Applied Computer Vision

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Lecture 8

Segmentation

Edge detection and boundary-based approaches: gradient, Laplacian, Laplacian of Gaussian, Canny, boundary chain codes, contour extraction, snakes

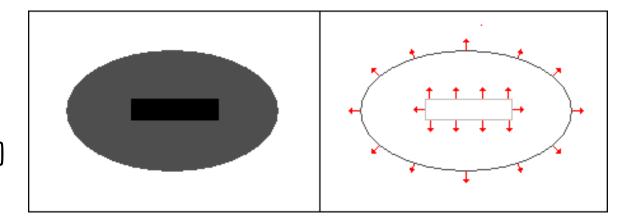
- An approach to segmentation
- Based on the analysis of the discontinuities in an image



Credit: Kenneth Dawson-Howe, A Practical Introduction to Computer Vision with OpenCV, © Wiley & Sons Inc. 2014

Edges have

- Magnitude
- Direction (Orientation)

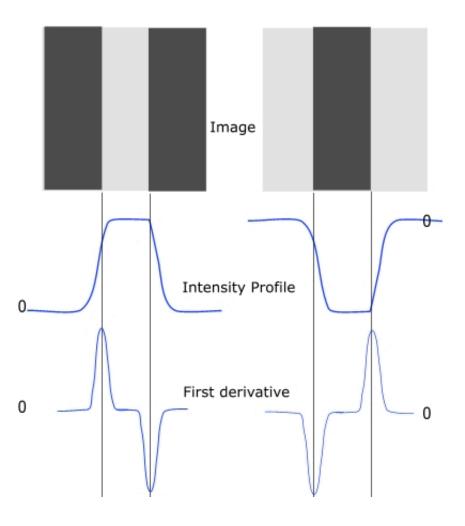


Edge Profiles

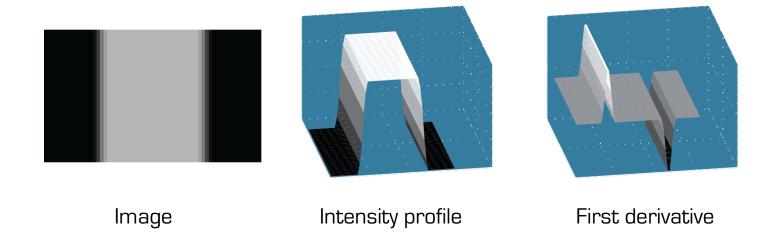
- Step
- Real
- Noisy



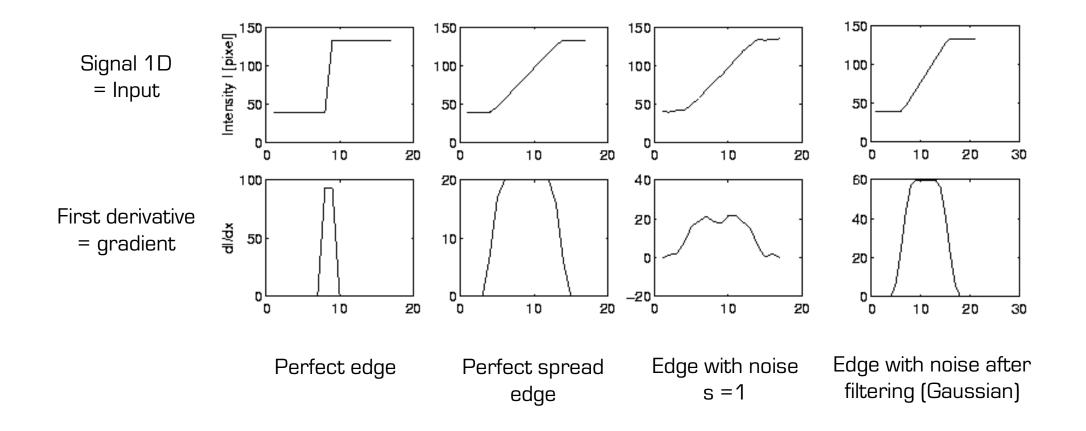
Credit: Kenneth Dawson-Howe, A Practical Introduction to Computer Vision with OpenCV, © Wiley & Sons Inc. 2014



Source: https://mipav.cit.nih.gov/pubwiki/index.php/Edge_Detection:_Zero_X_Laplacian



- Define a local edge in an image to be a transition between two regions of significantly different intensities
- The gradient function of the image, which measures the rate of change, will have large values in these transitional boundary areas
 - Enhance the image by estimating its gradient function
 - An edge is present if the gradient value is greater than some defined threshold



Credit: Markus Vincze, Technische Universität Wien

 Gradient functions are easy to understand in the discrete domain of digital images

Derivatives become simple first differences (h = 1)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Thus, the first difference of a 1D function is simply

$$f(x+1) - f(x)$$

Consider the following

1-D discrete (sampled & quantized) signal f(x)

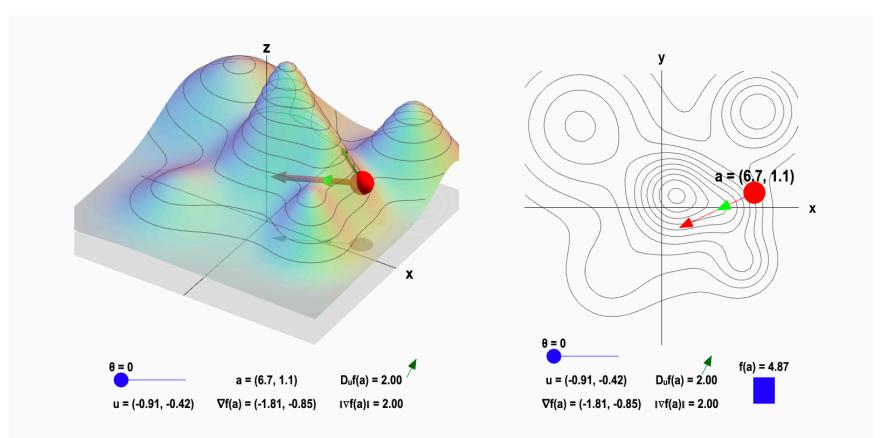
its first derivative (i.e. 1^{st} difference) f'(x)

f(x) 1 2 2 1 0 1 1 0 1 9 8 9 9 9 8

f'(x) 1 0 -1 -1 1 0 -1 1 8 -1 1 0 0 -1

In a 2D image, the gradient is a vector

It has a magnitude and a direction



Gradient and directional derivative on a mountain. The height of a mountain range described by a function f(x, y) is shown as surface plot in three-dimensions (left) and a two-dimensional level curve plot (right). In each panel, a red point can be moved by the mouse to change the location \mathbf{a} where the gradient $\nabla f(\mathbf{a})$ is calculated. Since f is a function of two variables, the point \mathbf{a} and the gradient are two-dimensional. The two-dimensional point \mathbf{a} is illustrated by the shadow of the red point on the xy-plane below the surface plot and by the red point itself on the level curve plot. The two dimensional gradient vector $\nabla f(\mathbf{a})$ is illustrated by the red vector emanating from the red point as well as by its shadow below the surface plot.

In a 2D image, the gradient is a vector

It has a magnitude and a direction

If $\frac{1}{2}$ and $\frac{1}{2}$ represent the rates of change of a 2D function f(x, y) in the x and y directions respectively, then the rate of change in a direction θ is given by

$$\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$$

• The direction θ at which the rate of change has the greatest magnitude is given by

$$\arctan \left[\frac{\partial}{\partial y} / \frac{\partial}{\partial x} \right]$$

- The magnitude is given by $\sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$
- The gradient of f(x, y) is a **vector** at (x, y) with this magnitude and direction

 Gradient functions are easy to understand in the discrete domain of digital images

Derivatives become simple first differences (h = 1)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

• For example, the first difference of a 2D function in the \boldsymbol{x} direction is simply

$$f(x+1, y) - f(x, y)$$

• Similarly, the first difference of a 2D function in the y direction is simply

$$f(x, y+1) - f(x, y)$$

The essential differences between all gradient edge detectors are

- the directions which the operators use to estimate the partial derivatives
- the manner in which they approximate the one-dimensional derivatives of the image function in these directions
- the manner in which they combine these approximations to form the gradient magnitude

(a) Roberts

1	0
0	-1

0 1 -1 0

(a)

(b) Sobel

-1	-2	-1
0	0	0
1	2	1

-1 0 1 -2 0 2 -1 0 1

(b)

(c) Prewitt

-1	-1	-1
0	0	0
1	1	1

-1 0 1 -1 0 1 -1 0 1

Roberts Cross

Strength: $G = \sqrt{G_x^2 + G_y^2}$, $G = |G_x| + |G_y|$

Angle: $\theta = \arctan(G_v / G_x) - 3\pi / 4$

+1	0
0	-1
\sim	OT / O



Original image



Edge strength



Binary image, threshold:80

Credit: Markus Vincze, Technische Universität Wien

Edge strength:
$$G = \sqrt{G_x^2 + G_y^2}$$

Angle: $\theta = \arctan(G_y / G_x)$

Sobel

-1	0	+1
-2	0	+2
-1	0	+1

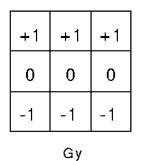
Gx

+1	+2	+1		
0	0	0		
-1	-2	-1		
Gy				

Prewitt

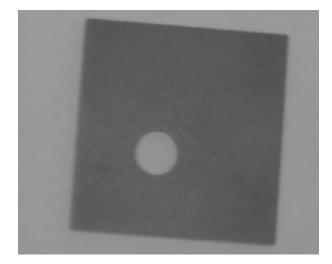
-1	0	+1
-1	0	+1
-1	0	+1

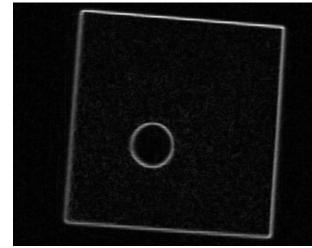
Gx





Credit: Markus Vincze, Technische Universität Wien



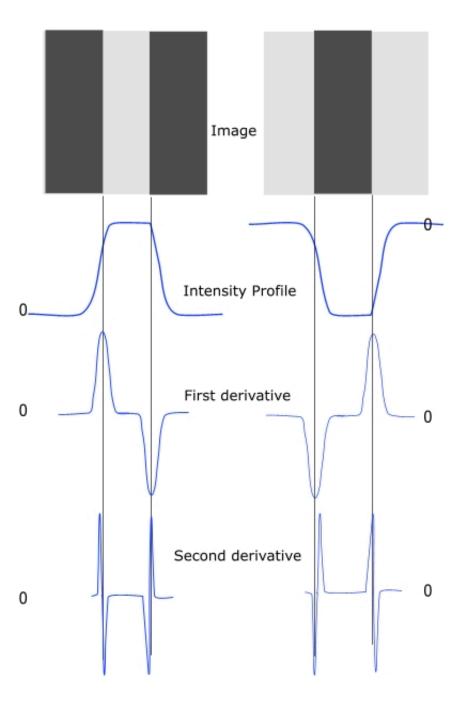


Edge detection has been discussed on the basis of first derivative directional operators

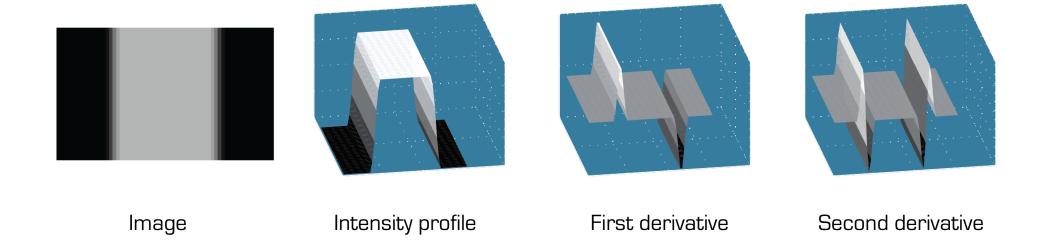
However, an alternative method uses an approximation to the Laplacian

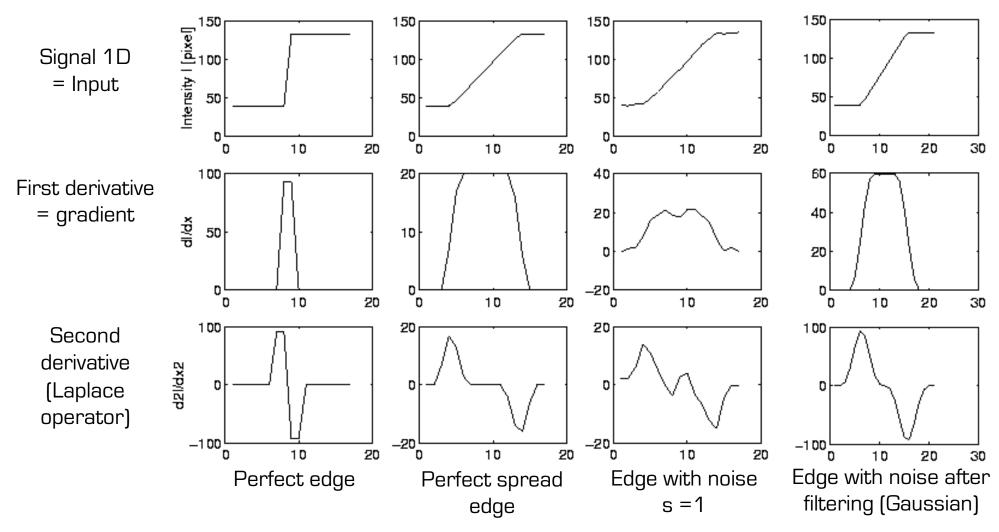
$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

i.e. the sum of second-order, unmixed, partial derivatives



Credit: https://mipav.cit.nih.gov/pubwiki/index.php/Edge_Detection:_Zero_X_Laplacian





Credit: Markus Vincze, Technische Universität Wien

 Gradient functions are easy to understand in the discrete domain of digital images

Derivatives become simple first differences (h = 1)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Thus, the first difference of a 1D function is simply

$$f(x+1) - f(x)$$

• The second difference of a 1D function is simply the first difference of the first difference

$$f(x+1) - 2f(x) + f(x-1)$$

$$1^{st}$$
 difference at x

$$f(x) - f(x-1)$$

1st difference at
$$x+1$$

$$f(x+1) - f(x)$$

 2^{nd} difference in x =

$$(f(x+1) - f(x)) - (f(x) - f(x-1))$$
= $f(x+1) - 2f(x) + f(x-1)$

This is often represented by a convolution or filter kernel

$$+1$$
 -2 $+1$

Consider the following

1-D discrete (sampled & quantized) signal f(x)

its first derivative (i.e. 1^{st} difference) f'(x)

its second derivative (i.e. 2^{nd} difference) f''(x)

f(x) 1 2 2 1 0 1 1 0 1 9 8 9 9 9 8

f'(x) 1 0 -1 -1 1 0 -1 1 8 -1 1 0 0 -1

f''(x) -1 -1 0 2 -1 -1 2 7 -9 2 -1 0 -1

The standard 2d approximation is given by:

$$L(x, y) = -4 f(x, y) + f(x-1, y) + f(x+1, y) + f(x, y-1) + f(x, y+1)$$

0	1	0
1	-4	1
0	1	0

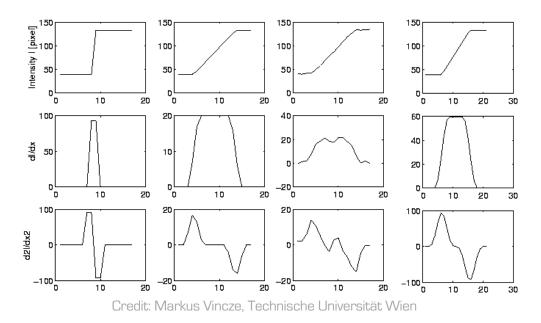
```
1st difference at x f(x,y) - f(x-1,y)
1st difference at x+1 f(x+1,y) - f(x,y)
2nd difference in x=
1st difference of 1st difference, i.e. (f(x+1,y) - f(x,y)) - (f(x,y) - f(x-1,y))
2nd difference in x= f(x+1,y) - 2f(x,y) + f(x-1,y)
2nd difference in y= f(x,y+1) - 2f(x,y) + f(x,y-1)
Laplacian = sum of 2nd differences = -4f(x,y) + f(x-1,y) + f(x+1,y) + f(x,y-1) + f(x,y+1)
```

The digital Laplacian has

- zero-response to linear ramps (and thus gradual changes in intensity)
- but it does respond on either side of the edge, once with a positive sign and once with a negative sign

To detect edges

- the image is enhanced by evaluating the digital Laplacian
- isolating the points at which the resultant image goes from positive to negative, i.e., at which it crosses zero



Marr-Hildreth edge detector (Laplacian of Gaussian)

- First smooth the image by convolving it with a two-dimensional Gaussian function
- and then isolate the zero-crossings of the Laplacian of this image

$$\nabla^2 \left\{ I(x,y) * G(x,y) \right\}$$

Why do we convolve the image with a Gaussian function?

- The Gaussian blurs the image
- This wipes out all structure at scales much smaller than the space constant σ (standard deviation) of the Gaussian
- Thus, we can select the spatial scale at which we process the image

Why the Gaussian?

- The Gaussian has the desirable characteristic of being smooth and localized in both the spatial and frequency domains
- It is the unique distribution that is simultaneously optimally localized in both domains
- Thus it is least likely to introduce any changes that were now present in the original image

The evaluation of the Laplacian and the convolution commute so that, for a Gaussian with a given standard deviation, we can derive a single filter

The Laplacian of Gaussian

$$\nabla^2 \left\{ I(x,y) * G(x,y) \right\} = \nabla^2 G(x,y) * I(x,y)$$

Furthermore, this 2D convolution is separable into four 1D convolutions

$$\nabla^{2} \left\{ I(x,y) * G(x,y) \right\} =$$

$$G(x) * \left\{ I(x,y) * \frac{\partial^{2}}{\partial y^{2}} G(y) \right\} + G(y) * \left\{ I(x,y) * \frac{\partial^{2}}{\partial x^{2}} G(x) \right\}$$

Laplacian

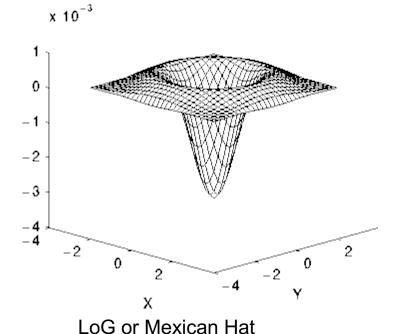
$$L(x,y) = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

Laplacian of Gaussian

$$LoG(x,y) = -\frac{1}{\pi\sigma^4} \left(1 - \frac{x^2 + y^2}{2\sigma^2} \right) e^{-\frac{x^2 + y^2}{2\sigma^2}}$$



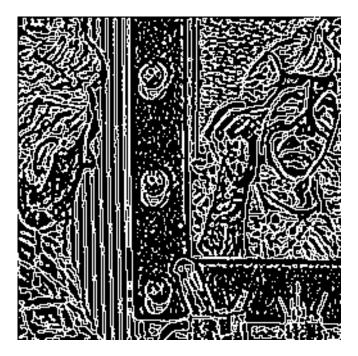
0	1	1	2	2	2	1	1	0
1	2	4	5	5	ю	4	2	1
1	4	5	3	0	Э	5	4	1
2	5	თ	-12	-24	-12	თ	5	2
2	5	0	-24	-40	-24	0	5	2
2	5	ო	-12	-24	-12	ო	5	2
1	4	5	3	0	э	15	4	1
1	2	4	5	5	5	4	2	1
0	1	1	2	2	2	1	1	0

Example filter 9x9, σ =1.4

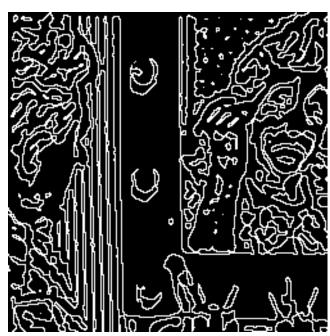
Credit: Markus Vincze, Technische Universität Wien



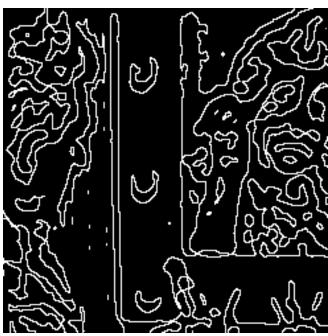
LoG with σ = 1



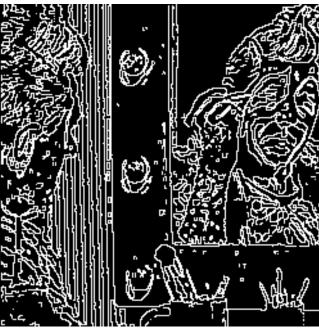
All zero crossings



Zero crossings at σ = 2



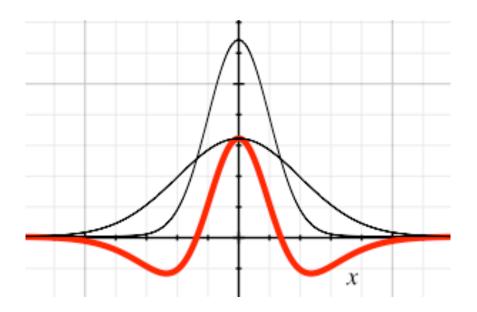
Zero crossings at σ = 3



 σ = 1, strong zero crossings (difference to neighbours >40)

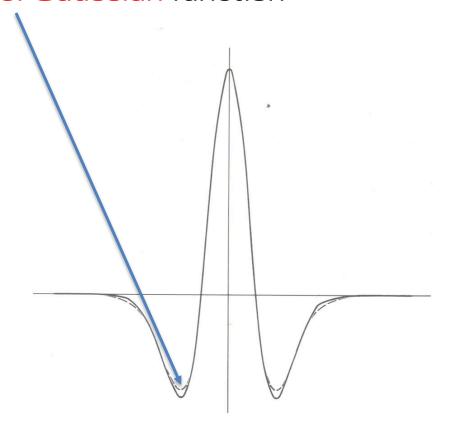
The Laplacian of Gaussian function is well approximated by the Difference of Gaussian function

Difference of two different Gaussian functions: $\sigma_1 = 1.6 \sigma_2$



Credit: http://bigwww.epfl.ch/teaching/iplablibrary/filtering4/index.php

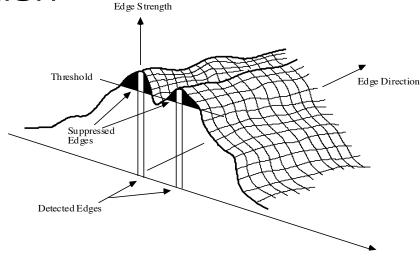
The Laplacian of Gaussian function is well approximated by the Difference of Gaussian function

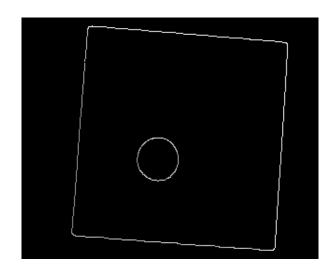


Credit: D. Marr, Vision, Freeman Press, 1982

Canny Edge Detector

- Gaussian smoothing
- 2. Gradient estimation
- 3. Ridge following with non-maxima suppression and hysteresis $(t_2>t_1)$
- Optimised, standard method
- Good compromise
- Thin, one (1) pixel edge (ridge)
- Smoothing eliminates detail

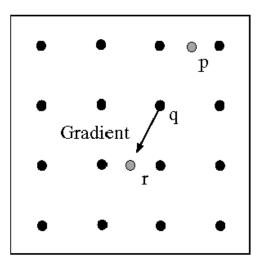




$$\sigma = 1, t_2 = 255, t_1 = 1$$

Canny Edge Detector

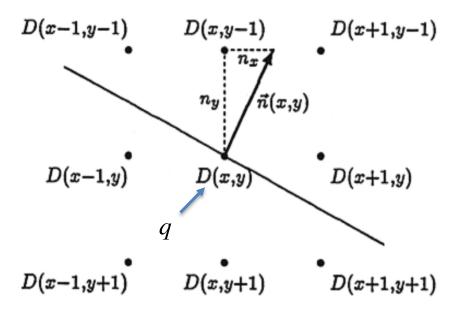
- Identify ridge pixels: maxima in gradient direction
- Value q is a maximum if larger than both p and r
- Interpolate to find p and r



Credit: Markus Vincze, Technische Universität Wien

Equations for finding the maxima

Notation of pixels:



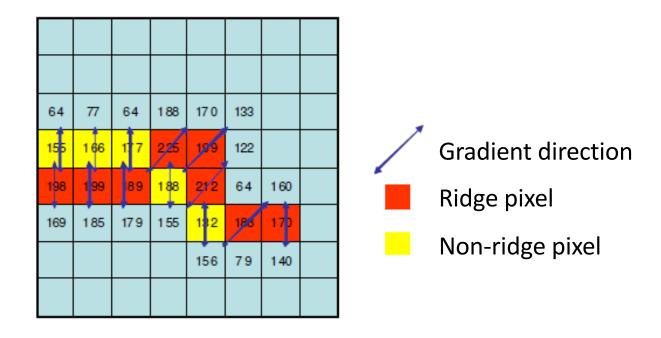
Interpolation equations:

$$p D_1 = \frac{n_x}{n_y} D(x+1, y-1) + \frac{n_y - n_x}{n_y} D(x, y-1)$$

$$D_2 = \frac{n_x}{n_y} D(x-1, y+1) + \frac{n_y - n_x}{n_y} D(x, y+1)$$

Canny edge detector

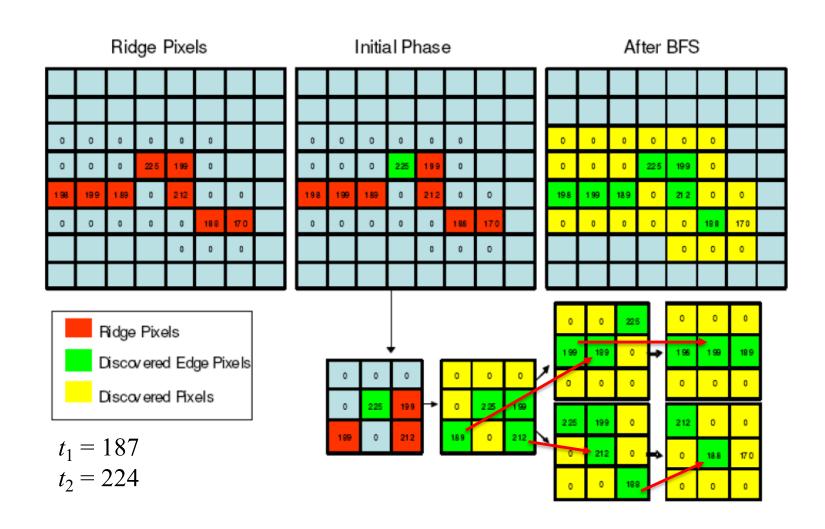
non-maxima suppression



Credit: Markus Vincze, Vienna University of Technology

Hysteresis Thresholding

- 2 thresholds for filtering edges:
 - Low threshold t_1
 - High threshold $t_2 > t_1$
- For all ridge pixels with magnitude $m \ge t_2$: mark as edge pixel
- $-t_1 < m < t_2$ and adjacent to an edge pixel: mark as edge pixel
- Identifies strong edges and eliminate weak edges with $m < t_1$



Hysteresis Algorithm

- Identify and mark all ridge pixels having $m \ge t_2$ as visited edges
- Add pixel into queue Q
- Run a breadth first search (BFS) on Q
 - For each pixel i in Q
 - For each unvisited adjacent pixel j of i
 - » Mark j as visited
 - » If $m(j) > t_1$, mark j as an edge and add j to Q
 - Remove i from Q
 - Terminate when Q is empty

Canny Edge Detector

- Y-Effect: 3 edges meeting in a point are not connected
- Adaptive: detail and edge elements, but image dependent



$$\sigma$$
 = 1, t_2 = 255, t_1 =1



$$\sigma$$
 = 1, t_2 = 255, t_2 = 220



 σ = 2, t_2 = 255, t_1 = 1

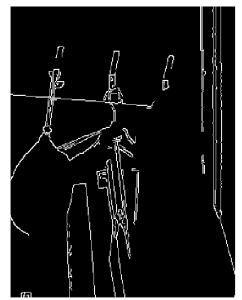
Credit: Markus Vincze, Vienna University of Technology

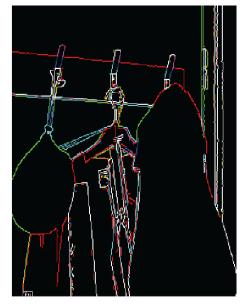
Multi-spectral edge detection

- Detect edges separately in each spectral band
- Use maximal value OR some linear combination.
- Use a different colour model
 - e.g. HLS space with linear combination of H & L









Credit: Kenneth Dawson-Howe, A Practical Introduction to Computer Vision with OpenCV, © Wiley & Sons Inc. 2014

Boundary Detection

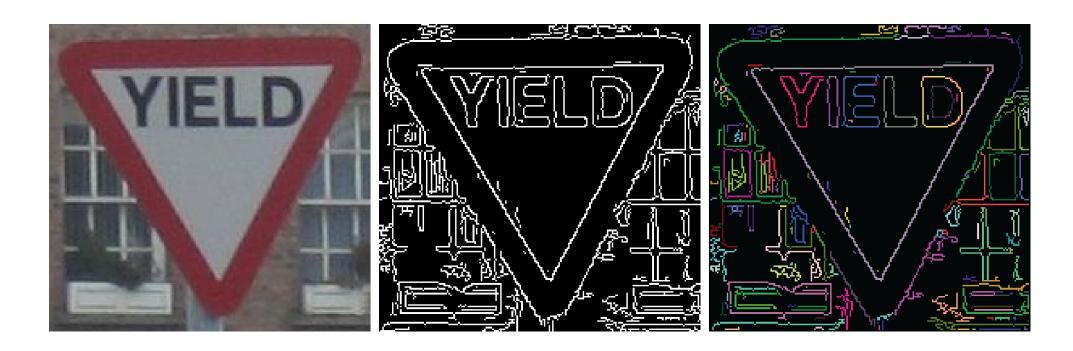
- Edge detection is just the first stage of the boundary-based segmentation process
- Also to aggregate these local edge elements
 - into structures better suited to the process of interpretation
- Normally achieved using processes such as
 - edge thinning (gradient-based edge operators produce thick edges)
 - edge linking
 - gap filling
 - curve-segment linking

Boundary Detection

- Techniques vary in the amount of knowledge or domaindependent information that is used in the grouping process
- In order of decreasing use of domain-dependent information
 - Boundary Refining
 - The Hough Transform
 - Graph Search
 - Dynamic Programming
 - Contour Following

Boundary Detection

Representation of Boundaries



In OpenCV, each individual contour is stored as a vector of points and all the contours are stored as a vector of contours (i.e. a vector of vector of points)

Credit: Kenneth Dawson-Howe, A Practical Introduction to Computer Vision with OpenCV, © Wiley & Sons Inc. 2014

The following code is taken from the sobelEdgeDetection project in the lectures directory of the ACV repository

```
sobelEdgeDetection.h
sobelEdgeDetectionImplementation.cpp
sobelEdgeDetectionApplication.cpp
```

```
* function sobelEdgeDetection
 * Trackbar callback - threshold user input
void sobelEdgeDetection(int, void*) {
   extern Mat inputImage;
   extern int thresholdValue;
   extern char* magnitude_window_name;
   extern char* direction_window_name;
   extern char* edge_window_name;
   Mat greyscaleImage;
   Mat edgeImage;
   Mat horizontal partial derivative;
   Mat vertical partial derivative;
  Mat 12norm gradient;
   Mat orientation;
   if (inputImage.type() == CV 8UC3) { // colour image
      cvtColor(inputImage, greyscaleImage, CV BGR2GRAY);
   else {
     greyscaleImage = inputImage.clone();
                 * This code is provided as part of "A Practical Introduction to Computer Vision with OpenCV"
   * by Kenneth Dawson-Howe @ Wiley & Sons Inc. 2014. All rights reserved.
   */
   Sobel(greyscaleImage,horizontal_partial_derivative,CV_32F,1,0);
   Sobel(greyscaleImage, vertical partial derivative, CV 32F, 0, 1);
   cartToPolar(horizontal partial derivative,vertical partial derivative,l2norm_gradient,orientation);
  Mat 12norm gradient gray = convert 32bit image for display( 12norm gradient );
  Mat 12norm_gradient_mask, display_orientation;
  12norm_gradient.convertTo(12norm_gradient_mask,CV_8U);
  threshold(12norm_gradient_mask,edgeImage,thresholdValue,255,THRESH_BINARY); // DV thresholdValue edgeImage
   orientation.copyTo(display orientation, edgeImage);
  Mat orientation gray = convert 32bit image for display(display orientation, 0.0, 255.0/(2.0*PI));
   imshow(magnitude_window_name, 12norm_gradient_gray); // DV
   imshow(direction window name, orientation gray);
                                                     // DV
   imshow(edge window name, edgeImage);
                                                     // DV
```

The following code is taken from the laplacianOfGaussian project in the lectures directory of the ACV repository

```
laplacianOfGaussian.h
laplacianOfGaussianImplementation.cpp
laplacianOfGaussianApplication.cpp
```

```
* function laplacianOfGaussian
 * Trackbar callback - Gaussian standard deviation input from user
*/
void laplacianOfGaussian(int, void*) {
   extern Mat src;
   extern int gaussian std dev;
   extern char* log_window name;
   extern char* zc_window_name;
   extern char* mod zc window name;
   int filter size;
   filter size = gaussian std dev * 4 + 1; // multiplier must be even to ensure an odd filter size as required by OpenCV
                                        // this places an upper limit on gaussian_std_dev of 7 to ensure the filter size < 31
                                        // which is the maximum size for the Laplacian operator
* This code is provided as part of "A Practical Introduction to Computer Vision with OpenCV"
 * by Kenneth Dawson-Howe @ Wiley & Sons Inc. 2014. All rights reserved.
 */
  Mat horizontal partial derivative, vertical partial derivative;
   Mat laplacian;
   Mat blurred image1 gray;
  Mat abs_gradient;
  Mat the gradient;
   GaussianBlur(src,blurred_image1_gray,Size(filter_size,filter_size),(double)gaussian_std_dev); // David Vernon: changed Size() and sigma argume
   Laplacian(blurred image1 gray,laplacian,CV 32F,filter size);
                                                                                        // David Vernon: changed kernel argument from 3
   Mat zero crossings;
   Mat modulated zero crossings;
   FindZeroCrossings(laplacian,zero crossings);
   Sobel(blurred image1 gray, horizontal partial derivative, CV 32F, 1,0);
   Sobel(blurred image1 gray, vertical partial derivative, CV 32F,0,1);
   abs gradient = abs(horizontal partial derivative) + abs(vertical partial derivative);
   abs gradient.convertTo(the gradient,CV 8U);
   bitwise_and( the_gradient, zero_crossings, modulated_zero_crossings ); // David Vernon: put the zero-crossings modulated by image gradient in
   Mat laplacian_gray = convert_32bit_image_for_display( laplacian, 128.0 );
imshow(log window name, laplacian gray);
   imshow(zc_window_name, zero_crossings);
   imshow(mod zc window name, modulated zero crossings);
```

The following code is taken from the cannyEdgeDetection project in the lectures directory of the ACV repository

```
cannyEdgeDetection.h
cannyEdgeDetectionImplementation.cpp
cannyEdgeDetectionApplication.cpp
```

```
/*
 * CannyThreshold
 * Trackbar callback - Canny thresholds input with a ratio 1:3
 */
void CannyThreshold(int, void*)
   extern Mat src;
   extern Mat src_gray;
   extern Mat src_blur;
   extern Mat detected edges;
   extern int cannyThreshold;
   extern char* canny window name;
   extern int gaussian_std_dev;
   int ratio = 3;
   int kernel_size = 3;
   int filter_size;
   filter_size = gaussian_std_dev * 4 + 1; // multiplier must be even to ensure an odd filter size as required by OpenCV
                                            // this places an upper limit on gaussian_std_dev of 7 to ensure the filter size < 31
                                            // which is the maximum size for the Laplacian operator
   cvtColor(src, src_gray, CV_BGR2GRAY);
   GaussianBlur(src_gray, src_blur, Size(filter_size,filter_size), gaussian_std_dev);
   Canny( src_blur, detected_edges, cannyThreshold, cannyThreshold*ratio, kernel_size );
   imshow( canny window name, detected edges );
```

The following code is taken from the contour Extraction project in the lectures directory of the ACV repository

```
contourExtraction.h
contourExtractionImplementation.cpp
contourExtractionApplication.cpp
```

```
/*
 * ContourExtraction
 * Trackbar callback - Canny hysteresis thresholds input with a ratio 1:3 and Gaussian standard deviation
 */
void ContourExtraction(int, void*) {
   extern Mat src;
   extern Mat src_gray;
   extern Mat src blur;
   extern Mat detected edges;
   extern int cannyThreshold;
   extern char* canny_window_name;
   extern char* contour window name;
   extern int gaussian_std_dev;
   bool debug = true;
   int ratio = 3;
   int kernel size = 3;
   int filter_size;
   vector<vector<Point>> contours;
   vector<Vec4i> hierarchy;
   Mat thresholdedImage;
   filter size = gaussian std dev * 4 + 1; // multiplier must be even to ensure an odd filter size as required by OpenCV
                                            // this places an upper limit on gaussian_std_dev of 7 to ensure the filter size < 31
                                            // which is the maximum size for the Laplacian operator
   cvtColor(src, src gray, CV BGR2GRAY);
   GaussianBlur(src_gray, src_blur, Size(filter_size,filter_size), gaussian_std_dev);
   Canny( src blur, detected edges, cannyThreshold, cannyThreshold*ratio, kernel size );
```

```
Mat canny_edge_image_copy = detected_edges.clone();  // clone the edge image because findContours overwrites it

/* see http://docs.opencv.org/2.4/modules/imgproc/doc/structural_analysis_and_shape_descriptors.html#findcontours */
/* and http://docs.opencv.org/2.4/doc/tutorials/imgproc/shapedescriptors/find_contours/find_contours.html */
findContours(canny_edge_image_copy,contours,hierarchy,CV_RETR_TREE,CV_CHAIN_APPROX_NONE);

Mat contours_image = Mat::zeros(src.size(), CV_8UC3);  // draw the contours on a black background

for (int contour_number=0; (contour_number<(int)contours.size()); contour_number++) {
    Scalar colour( rand()&0xFF, rand()&0xFF, rand()&0xFF );  // use a random colour for each contour
    drawContours( contours_image, contours, contour_number, colour, 1, 8, hierarchy );
}

if (debug) printf("Number of contours %d: \n", contours.size());
imshow( canny_window_name, detected_edges );
imshow( contour_window_name, contours_image );</pre>
```

}

Exercises

- 1. Read OpenCV documentation for all OpenCV functions in sample code
- 2. Study utility functions in sample code
- 3. Replace Canny edge detection with Laplacian of Gaussian edge detection in contour Extraction. Comment on any differences you see

Reading

R. Szeliski, Computer Vision: Algorithms and Applications, Springer, 2010.

Section 4.2 Edges