# Applied Computer Vision

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### Lecture 11

Image Features

Harris and interest point operator

# Approaches to Object Recognition

### Generic Gestalt Principles

- The world is structured, extract features
- perceptual grouping

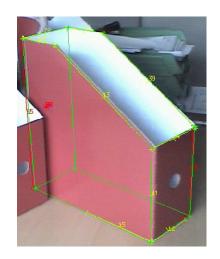


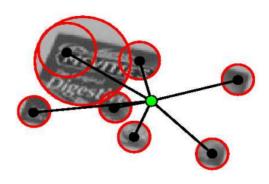
- CAD model of object
- Geometric features
- Locate features and their arrangement

### Appearance based

- Interest points / point features
- or "whole" object



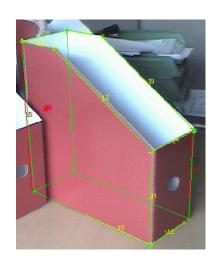


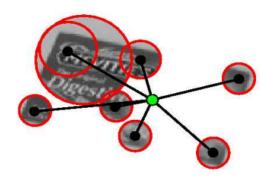


# Approaches to Object Recognition

- Point features
  - Key point features
  - Interest points
  - Corners
- Find features in one image and track
- Find features in all images and match







# Objects and Interest Points (IPs)

#### 1. Feature detection

Extract interest points (unique image regions)

2. Feature description
Calculate local
(invariant) descriptors

- 3. Feature matching / feature tracking Find correspondences
- 4. Find similar image regions/objects

# Example: Object Recognition for Image Stitching

- How to recognize the same objects or parts of objects in different images?
- Example: Panorama putting many images together

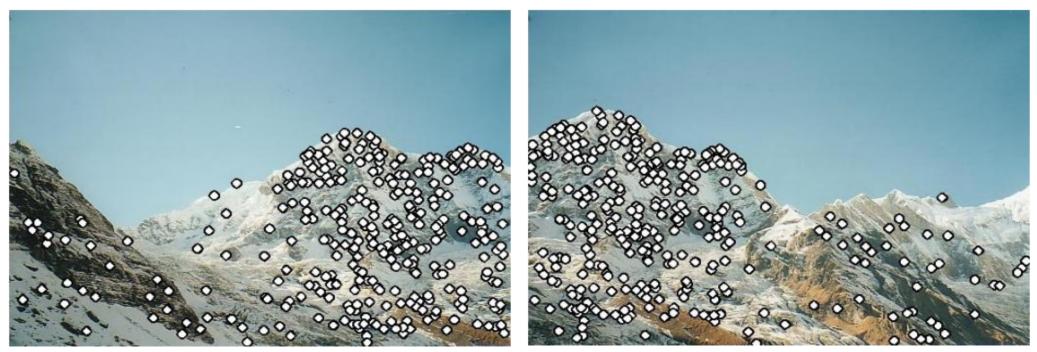




Credit: Markus Vincze, Technische Universität Wien

# Image Stitching: Interest Points (IPs)

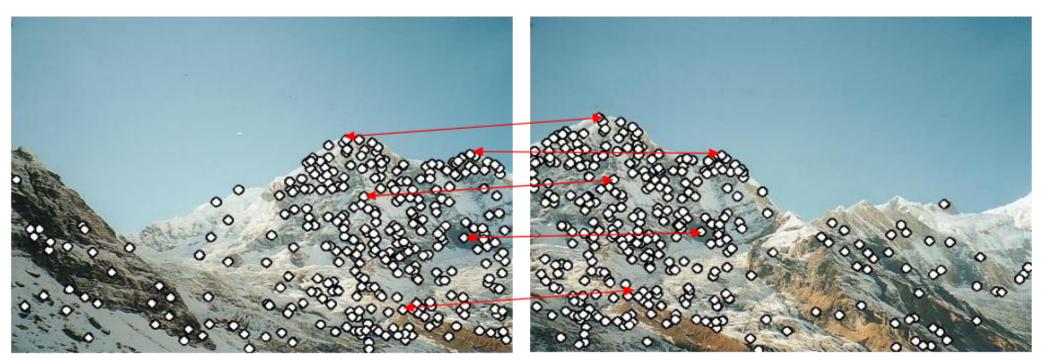
- Recognition of features in both images
- Finding of corresponding point pairs



Credit: Markus Vincze, Technische Universität Wien

# Image Stitching: Matching

- Recognition of features in both images
- Finding of corresponding point pairs
- Use points to put image together → panorama



Credit: Markus Vincze, Technische Universität Wien

# Image Stitching: Matching

- Recognition of features in both images
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Credit: Markus Vincze, Technische Universität Wien

# Matching with Features

#### Problem 1:

Detection of the same point pair independently in two images

- Here: not the same points, no match
- Need: reliable and distinctive point descriptor



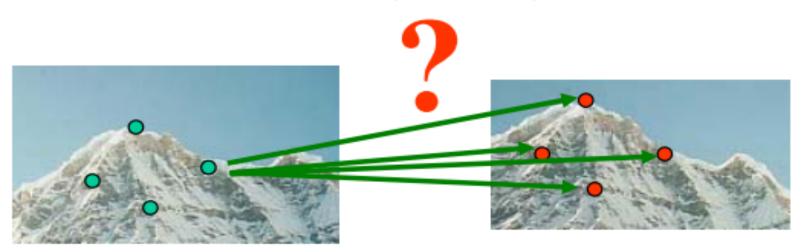


# Matching with Features

#### Problem 2:

For every point in the image, find the corresponding point in the other image

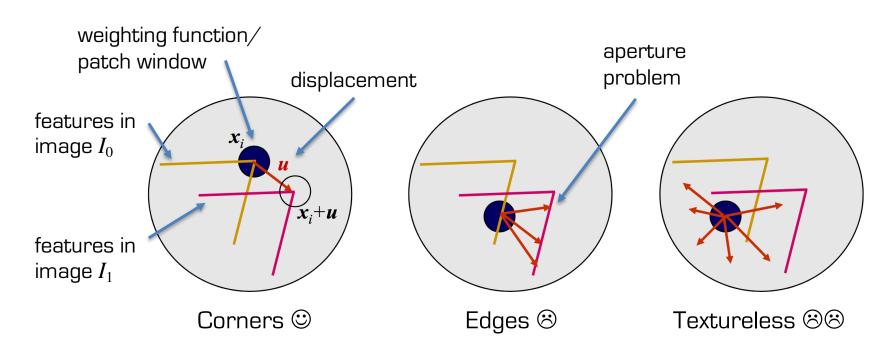
- Here: which point on the right is correct?
- Need: reliable and distinctive point descriptor



### Many different approaches

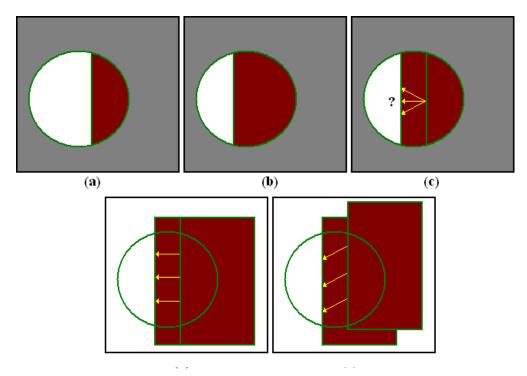
- Corner detector: Harris (1988), Hessian
- Multi-scale corner detector with scale selection
  - Scale invariant Harris and Hessian corners
  - Difference of Gaussian (DoG) (Lowe 2004)
- Affine covariant Regions
  - Harris-Affine (Mikolajczyk, Schmid '02, Schaffalitzky, Zisserman '02)
  - Hessian-Affine (Mikolajczyk and Schmid '02)
  - Maximally stable extremal regions (MSER) (Matas et al. '02)
  - Intensity based regions (IBR) (Tuytelaars and Van Gool '00)
  - Edge based regions (EBR) (Tuytelaars and Van Gool '00)
  - Entropy-based regions (salient regions) (Kadir et al. '04)
  - Features from accelerated segment test (FAST) (Rosten et al. '05)

- Textureless patches are almost impossible to localize
- Patches with high contrast (gradient) are easier to localize
- Straight-line segments suffer from the aperture problem



Credit: Szelisky 2010

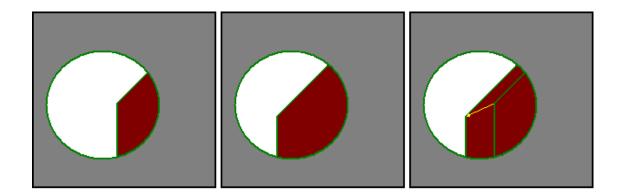
- Aperture problem with edges
- Given two images (a) and (b) taken at different times, determine the movement of edge points from frame 1 to frame 2 (c) ...



Credit: Kenneth Dawson-Howe, A Practical Introduction to Computer Vision with OpenCV, © Wiley & Sons Inc. 2014

Use corners / point features / interest points

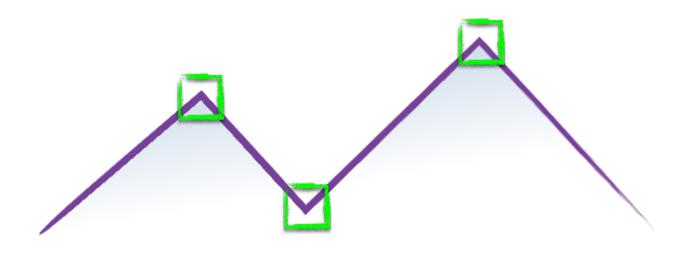
- corner ... intersection of two edges
- interest point ... any feature that can be robustly detected



Credit: Kenneth Dawson-Howe, A Practical Introduction to Computer Vision with OpenCV, © Wiley & Sons Inc. 2014

# How do you find a corner?

[Moravec 1980]

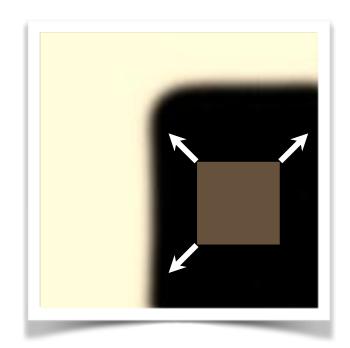


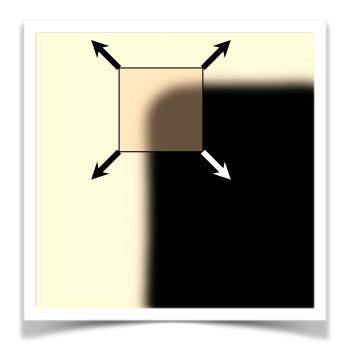
Easily recognized by looking through a small window

Shifting (displacing) the window should give large change in intensity

#### Easily recognized by looking through a small window

#### Shifting (displacing) the window should give large change in intensity





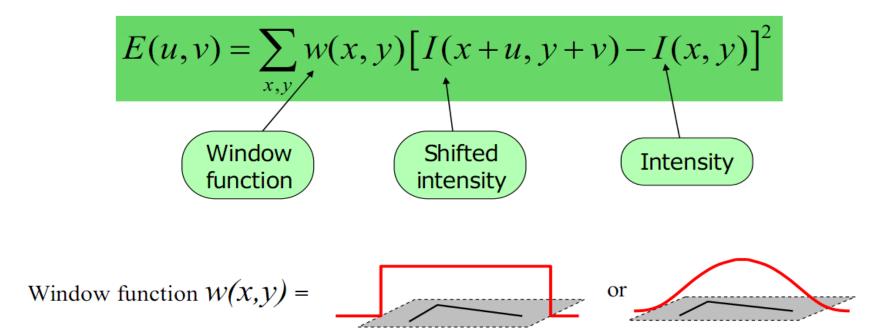
"flat" region: no change in all directions

"edge":
no change along the edge
direction

"corner": significant change in all directions

#### Autocorrelation function

How well an image patch matches itself as a function of a small displacement (sum of squared differences in a region window)



Credit: Markus Vincze, Technische Universität Wien

1 in window, 0 outside

Gaussian

For small shifts [u, v] we have a bilinear approximation

$$E(u,v) \cong \begin{bmatrix} u,v \end{bmatrix} \quad M \quad \begin{bmatrix} u\\v \end{bmatrix}$$

where M is the  $2\times2$  autocorrelation matrix computed from image derivatives  $I_x$  and  $I_x$ 

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

To see why, let's consider the following

Also known as the covariance matrix

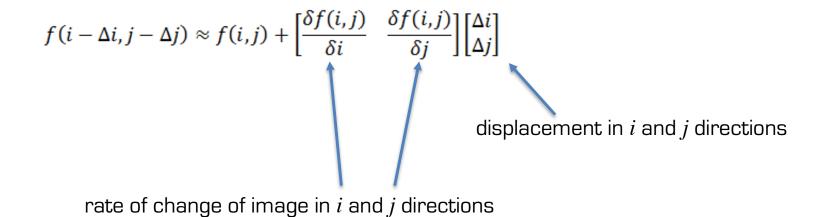
Consider the intensity variation for a displacement  $(\Delta i, \Delta j)$  as sum of squared differences SSD

assuming a box weighting / windowing function w

$$SSD_{W}(\Delta i, \Delta j) = \sum_{(i,j) \in W} (f(i,j) - f(i - \Delta i, j - \Delta j))^{2}$$

Consider the intensity variation for a displacement  $(\Delta i, \Delta j)$  as sum of squared differences SSD

Approximating the displaced image as follows



Credit: Kenneth Dawson-Howe, A Practical Introduction to Computer Vision with OpenCV, © Wiley & Sons Inc. 2014

Consider the intensity variation for a displacement  $(\Delta i, \Delta j)$  as sum of squared differences SSD

Substituting terms and simplifying

$$SSD_{W}(\Delta i, \Delta j) = \sum_{(i,j) \in W} \left( f(i,j) - f(i,j) - \left[ \frac{\delta f(i,j)}{\delta i} \quad \frac{\delta f(i,j)}{\delta j} \right] \begin{bmatrix} \Delta i \\ \Delta j \end{bmatrix} \right)^{2}$$

$$SSD_{W}(\Delta i, \Delta j) = \sum_{(i,j) \in W} \left( \left[ \frac{\delta f(i,j)}{\delta i} \quad \frac{\delta f(i,j)}{\delta j} \right] \left[ \frac{\Delta i}{\Delta j} \right] \right)^{2}$$

$$SSD_{W}(\Delta i, \Delta j) = \sum_{(i,j) \in W} \left( \begin{bmatrix} \Delta i & \Delta j \end{bmatrix} \left( \begin{bmatrix} \frac{\delta f(i,j)}{\delta i} \\ \frac{\delta f(i,j)}{\delta j} \end{bmatrix} \begin{bmatrix} \frac{\delta f(i,j)}{\delta i} & \frac{\delta f(i,j)}{\delta j} \end{bmatrix} \right) \begin{bmatrix} \Delta i \\ \Delta j \end{bmatrix} \right)$$

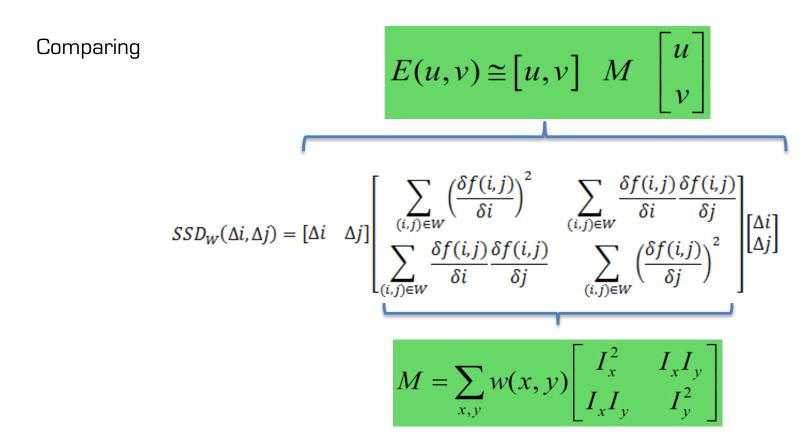
Credit: Kenneth Dawson-Howe, A Practical Introduction to Computer Vision with OpenCV, © Wiley & Sons Inc. 2014

Consider the intensity variation for a displacement  $(\Delta i, \Delta j)$  as sum of squared differences SSD

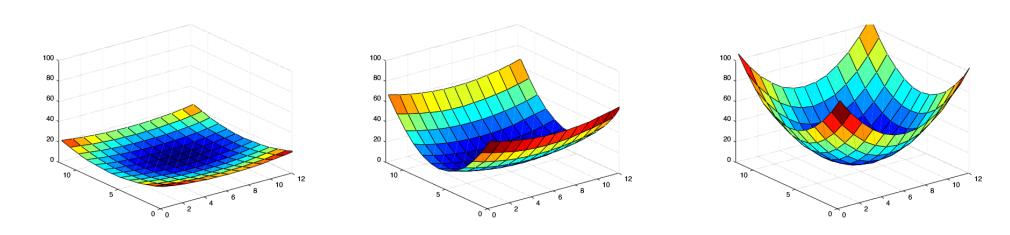
Rearranging

$$SSD_{W}(\Delta i, \Delta j) = \begin{bmatrix} \Delta i & \Delta j \end{bmatrix} \begin{bmatrix} \sum\limits_{(i,j) \in W} \left(\frac{\delta f(i,j)}{\delta i}\right)^{2} & \sum\limits_{(i,j) \in W} \frac{\delta f(i,j)}{\delta i} \frac{\delta f(i,j)}{\delta j} \\ \sum\limits_{(i,j) \in W} \frac{\delta f(i,j)}{\delta i} \frac{\delta f(i,j)}{\delta j} & \sum\limits_{(i,j) \in W} \left(\frac{\delta f(i,j)}{\delta j}\right)^{2} \end{bmatrix} \begin{bmatrix} \Delta i \\ \Delta j \end{bmatrix}$$

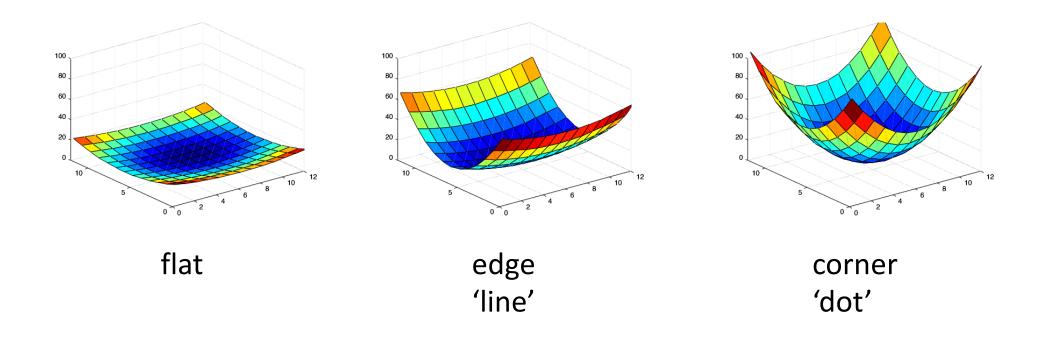
Consider the intensity variation for a displacement  $(\Delta i, \Delta j)$  as sum of squared differences SSD

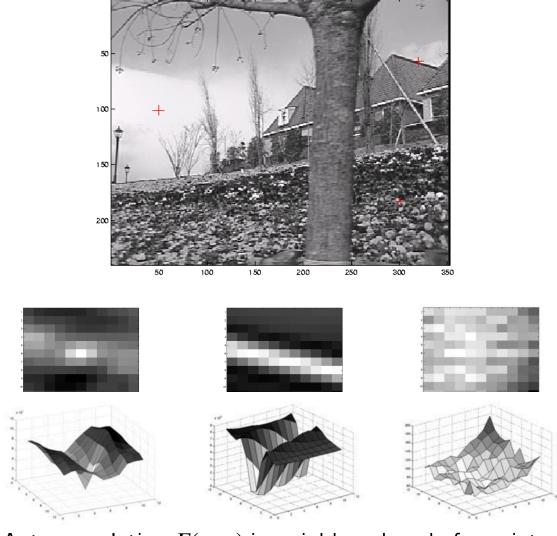


#### Which error surface indicates a good image feature?



What kind of image patch do these surfaces represent?



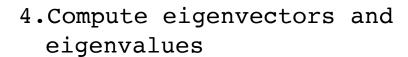


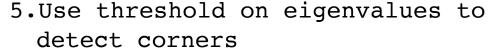
Autocorrelation E(u, v) in neighbourhood of a point

Credit: Szeliski 2010

- 1.Compute image gradients over small region
- 2.Subtract mean from each image gradient







$$I_x = \frac{\partial I}{\partial x}$$

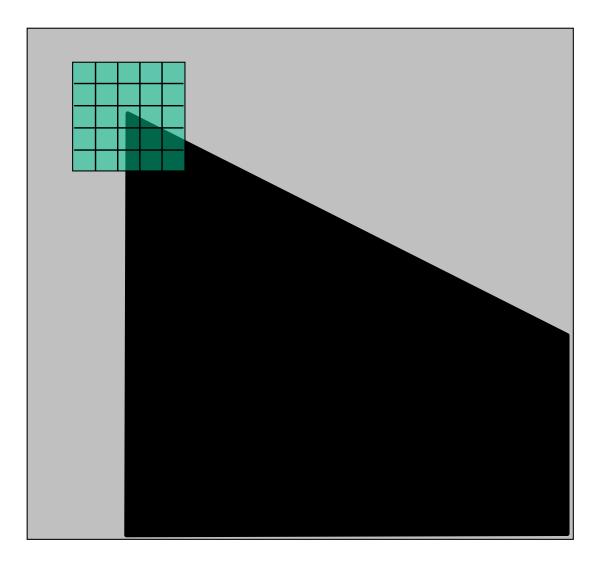


$$I_y = \frac{\partial I}{\partial y}$$



$$\left[\begin{array}{ccc} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{array}\right]$$

# 1. Compute image gradients over a small region (not just a single pixel)



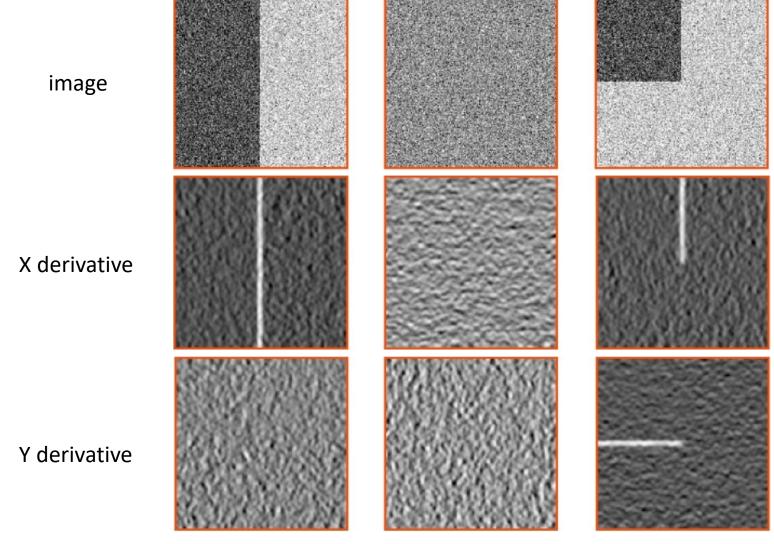
array of x gradients

$$T_x = \frac{\partial I}{\partial x}$$

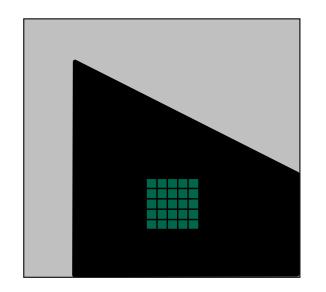
array of y gradients

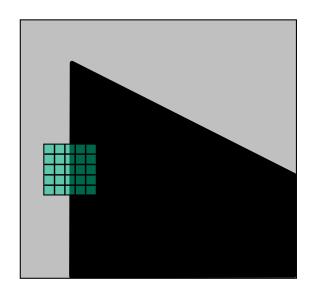
$$I_y = \frac{\partial I}{\partial y}$$

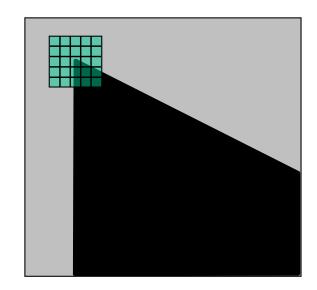
Credit: Kris Kitani, Carnegie Mellon University

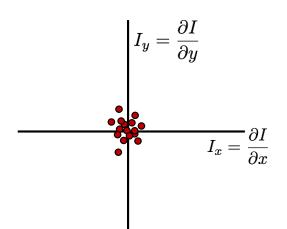


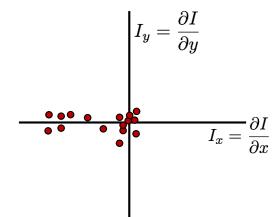
Credit: Kris Kitani, Carnegie Mellon University

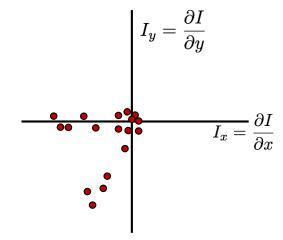




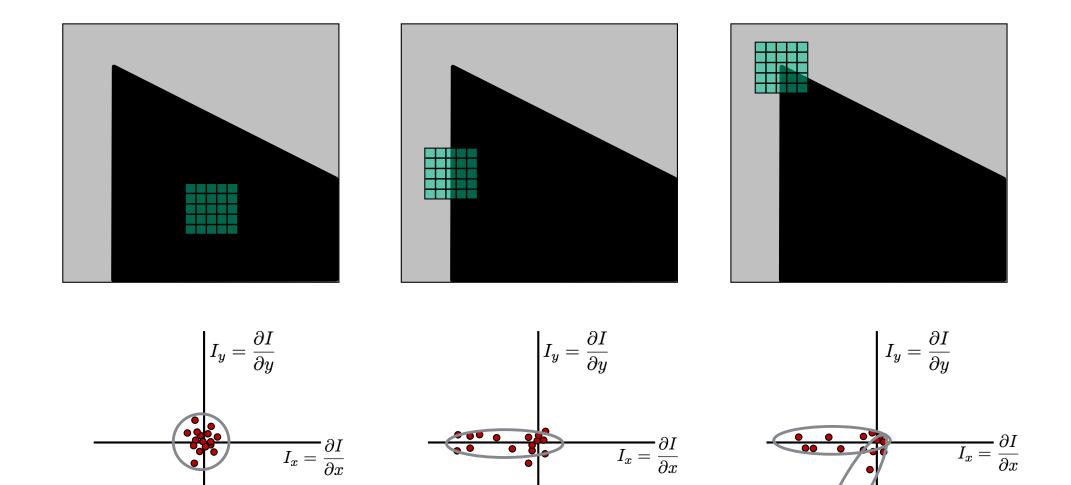




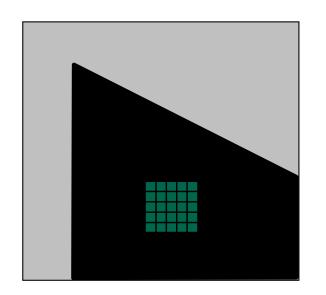


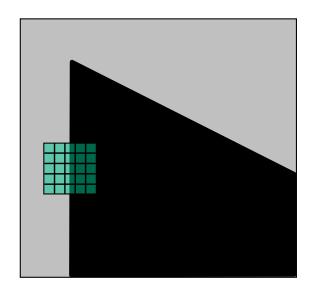


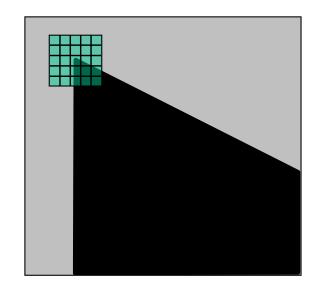
What does the distribution tell you about the region?

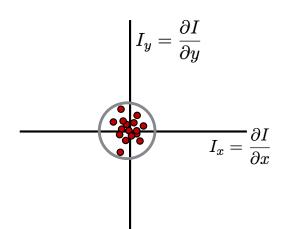


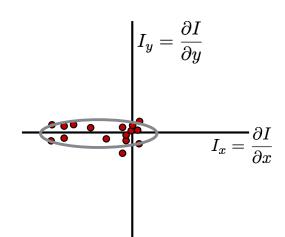
Distribution reveals edge orientation and magnitude

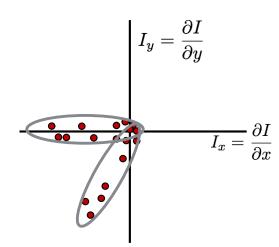








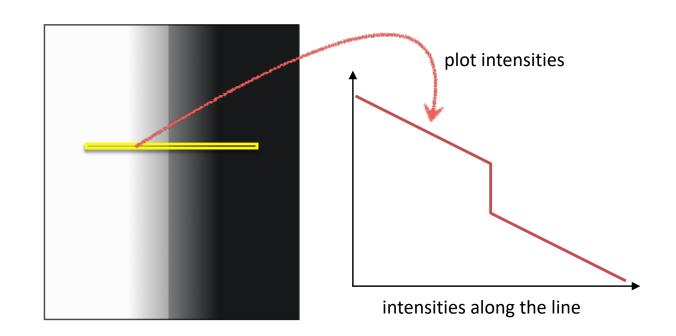




### How do you quantify orientation and magnitude?

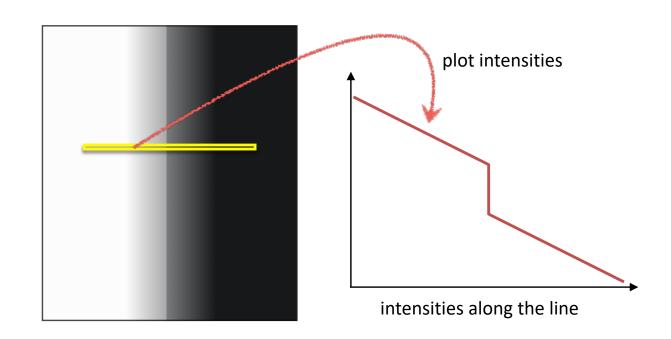
### 2. Subtract the mean from each image gradient

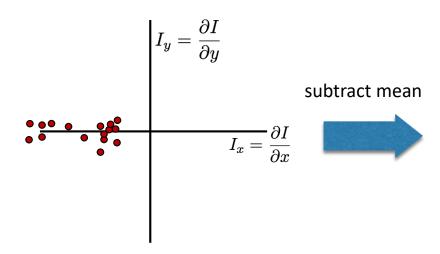
constant intensity gradient



### 2. Subtract the mean from each image gradient

constant intensity gradient

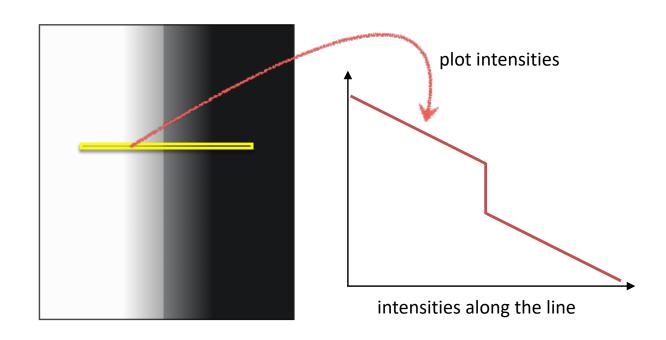


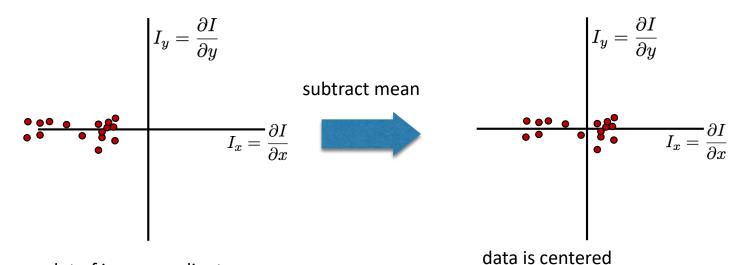


plot of image gradients

### 2. Subtract the mean from each image gradient

constant intensity gradient



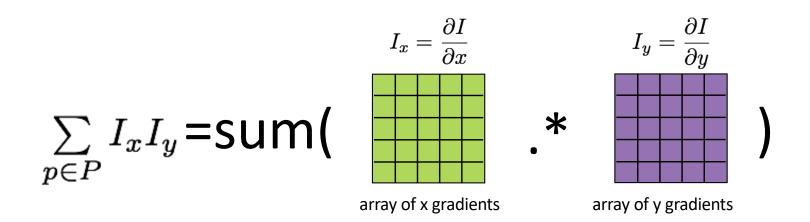


plot of image gradients

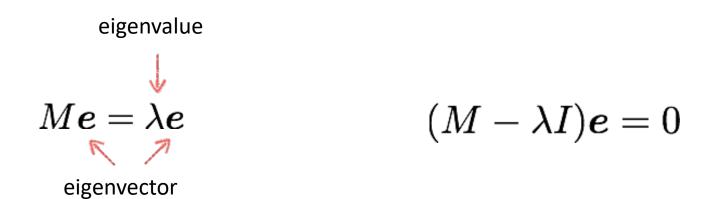
ts Credit: Kris Kitani, Carnegie Mellon Universit('DC' offset is removed)

### 3. Compute the covariance matrix

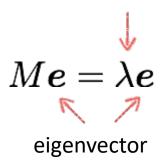
$$\left[\begin{array}{ccc} \sum\limits_{p\in P}I_xI_x & \sum\limits_{p\in P}I_xI_y \\ \sum\limits_{p\in P}I_yI_x & \sum\limits_{p\in P}I_yI_y \end{array}\right]$$



Where does this covariance matrix come from?



eigenvalue



$$(M - \lambda I)\mathbf{e} = 0$$

1. Compute the determinant of

(returns a polynomial)

$$M - \lambda I$$

#### eigenvalue

$$Moldsymbol{e}=\lambdaoldsymbol{e}$$
 eigenvector

$$(M - \lambda I)\mathbf{e} = 0$$

1. Compute the determinant of

(returns a polynomial)

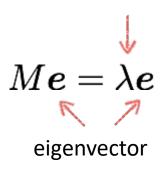
 $M - \lambda I$ 

2. Find the roots of polynomial

(returns eigenvalues)

$$\det(M - \lambda I) = 0$$

#### eigenvalue



$$(M - \lambda I)\mathbf{e} = 0$$

1. Compute the determinant of

(returns a polynomial)

 $M - \lambda I$ 

2. Find the roots of polynomial

(returns eigenvalues)

 $\det(M - \lambda I) = 0$ 

3. For each eigenvalue, solve

(returns eigenvectors)

$$(M - \lambda I)\mathbf{e} = 0$$

Credit: Kris Kitani, Carnegie Mellon University

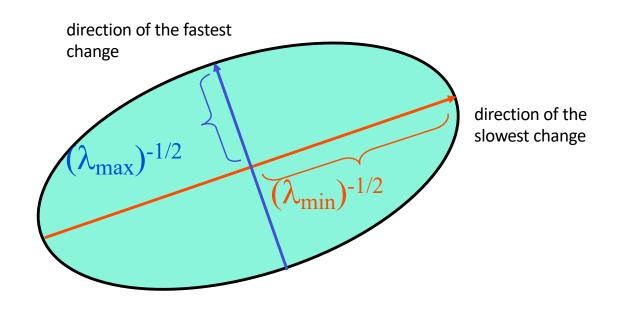
Since M is symmetric, we have

$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

We can visualize M as an ellipse with axis lengths determined by the eigenvalues and orientation determined by R

Ellipse equation:

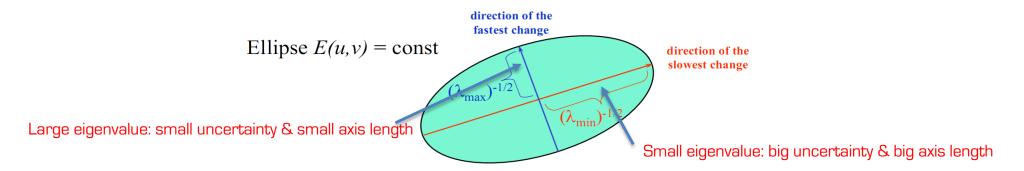
$$\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



Credit: Kris Kitani, Carnegie Mellon University

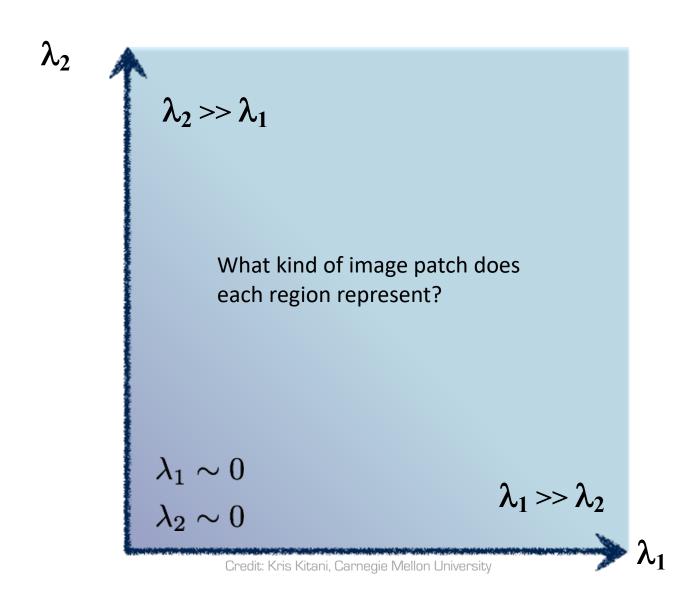
#### Eigenvalue analysis of M

- Eigenvalues  $\lambda_1$  and  $\lambda_2$  of M indicate maximum and minimum directions of gradient, respectively
- Larger uncertainty depends on the smaller eigenvalue
  - Find points where the value of the smaller eigenvalue is large (i.e. where both eigenvalues are large)
  - These indicate good features to track, i.e. corners

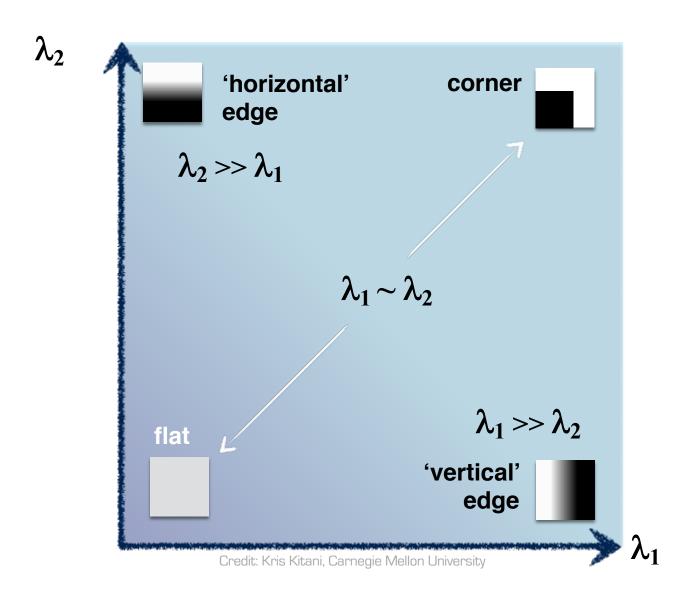


Credit: Markus Vincze, Technische Universität Wien

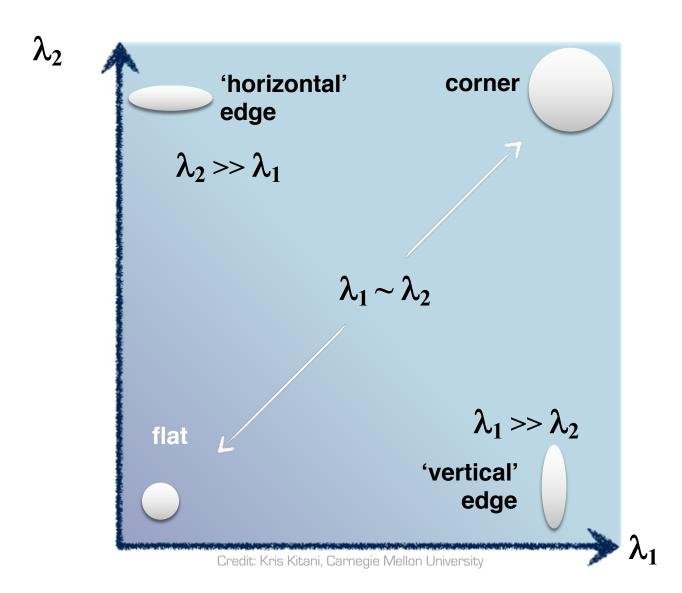
# Interpreting eigenvalues

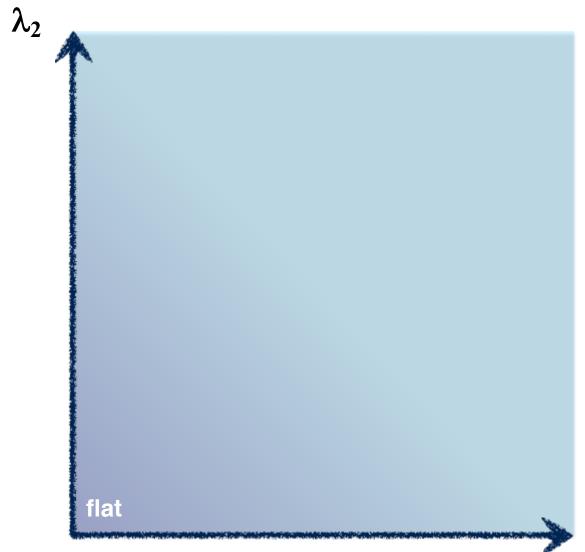


# Interpreting eigenvalues

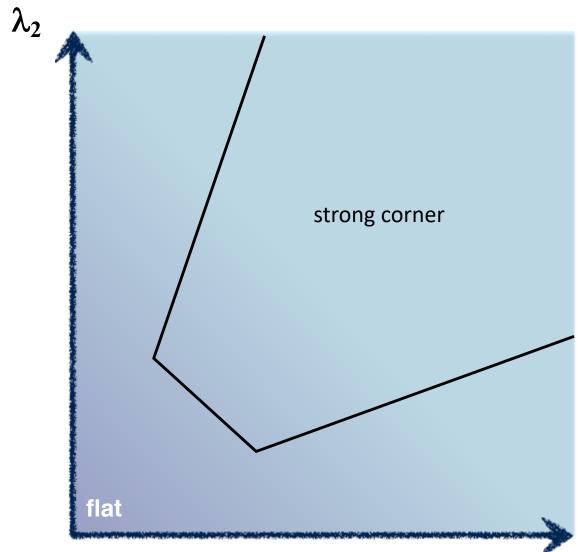


# Interpreting eigenvalues



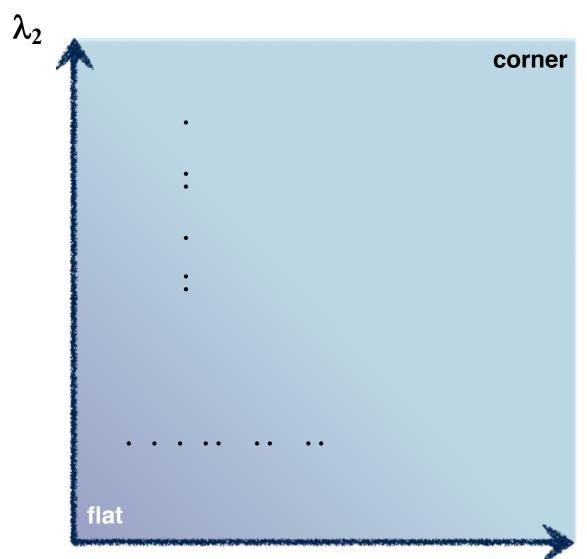


Think of a function to score 'cornerness'



Think of a function to score 'cornerness'

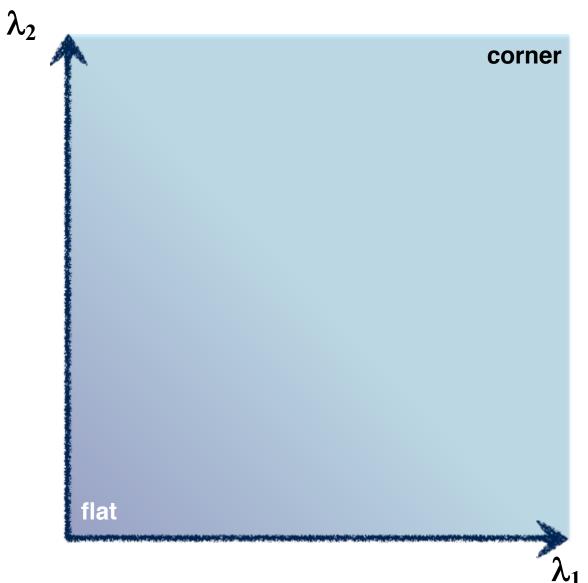
(a function of)



Use the smallest eigenvalue as the response function

$$R = \min(\lambda_1, \lambda_2)$$

(a function of)

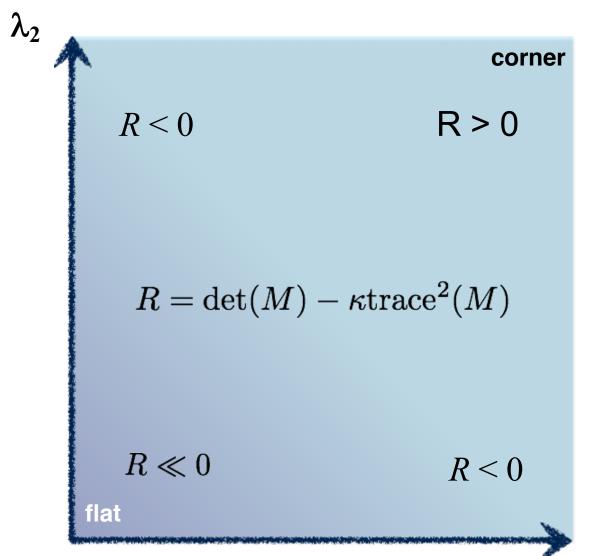


Eigenvalues need to be bigger than one.

$$R = \lambda_1 \lambda_2 - \kappa (\lambda_1 + \lambda_2)^2$$

Can compute this more efficiently...

(a function of)



$$\det M = \lambda_1 \lambda_2$$

$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

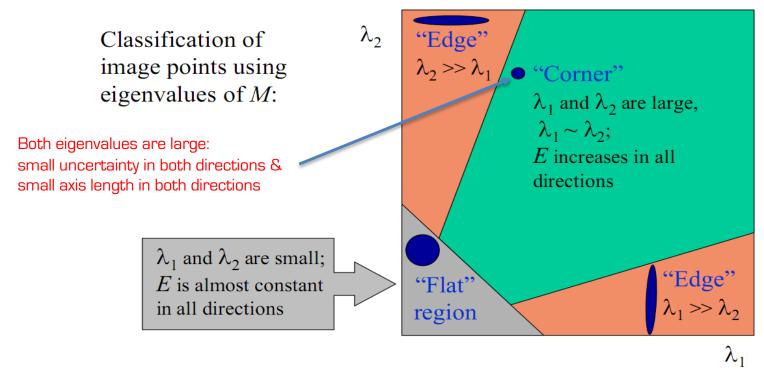
$$\det \begin{pmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \end{pmatrix} = ad - bc$$

$$\operatorname{trace} \begin{pmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \end{pmatrix} = a + d$$

# Summary

Intensity change in shifting window: eigenvalue analysis

Eigenvalues.  $\lambda_1$  and  $\lambda_2$  of M indicate max and min directions of gradient



Credit: Markus Vincze, Technische Universität Wien

# Summary

Measure of "cornerness" without computing eigenvalues explicitly:

Maximize det M

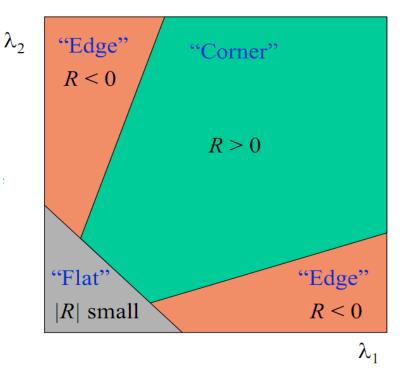
$$R = \det M - k \left( \operatorname{trace} M \right)^2$$

$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

(k - empirical constant, k = 0.04-0.06)

# Summary

- R is negative for edges
- R is small for flat regions
- Find points with large measure of cornerness (R > threshold)
- Take points that are local maxima in R



#### Harris & Stephens (1988)

$$R = \det(M) - \kappa \operatorname{trace}^2(M)$$

#### Kanade & Tomasi (1994)

$$R = \min(\lambda_1, \lambda_2)$$

#### Nobel (1998)

$$R = \frac{\det(M)}{\operatorname{trace}(M) + \epsilon}$$

Credit: Kris Kitani, Carnegie Mellon University

### Harris Detector

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." 1988.

1. Compute x and y derivatives of image

$$I_{x} = G_{\sigma}^{x} * I \qquad I_{y} = G_{\sigma}^{y} * I$$

2. Compute products of derivatives at every pixel

$$I_{x^2} = I_x \cdot I_x$$
  $I_{y^2} = I_y \cdot I_y$   $I_{xy} = I_x \cdot I_y$ 

Compute the sums of the products of derivatives at each pixel

$$S_{x^2} = G_{\sigma'} * I_{x^2}$$
  $S_{y^2} = G_{\sigma'} * I_{y^2}$   $S_{xy} = G_{\sigma'} * I_{xy}$ 

### Harris Detector

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." 1988.

4. Define the matrix at each pixel

$$M(x,y) = \begin{bmatrix} S_{x^2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y^2}(x,y) \end{bmatrix}$$

5. Compute the response of the detector at each pixel

$$R = \det M - k (\operatorname{trace} M)^2$$

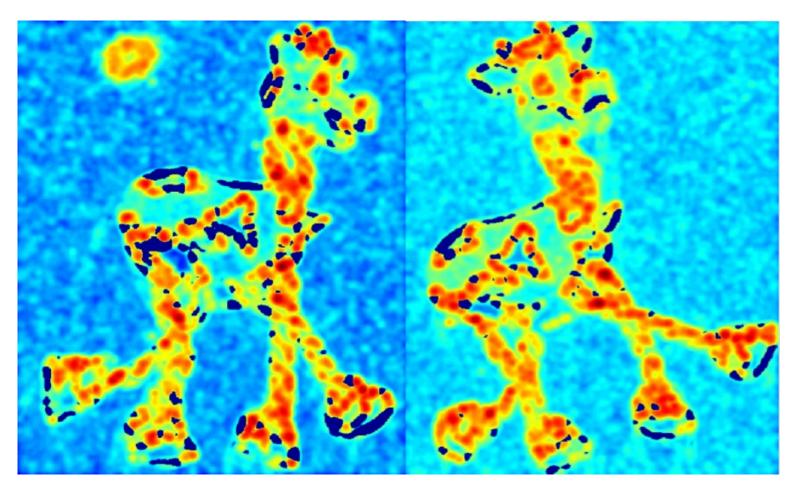
6. Threshold on value of R; compute non-max suppression.

### Same object with different illumination and pose



Credit: Markus Vincze, Technische Universität Wien

#### R values



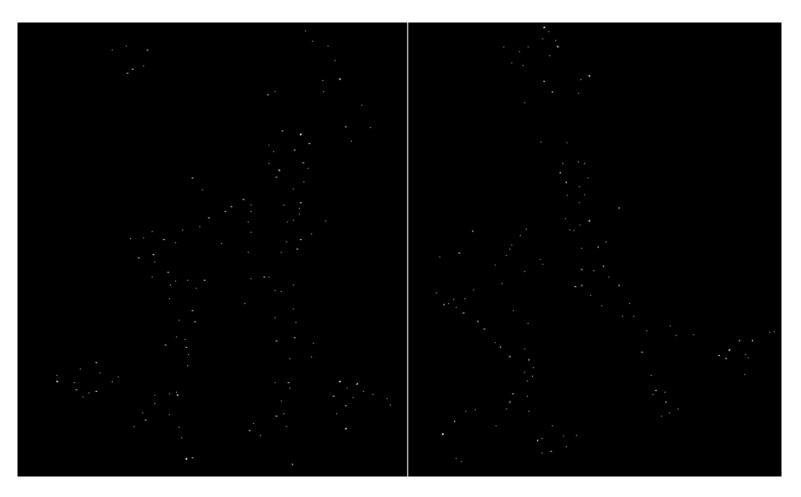
Credit: Markus Vincze, Technische Universität Wien

Points (regions) with R larger than a threshold



Credit: Markus Vincze, Technische Universität Wien

#### Local maxima



Credit: Markus Vincze, Technische Universität Wien

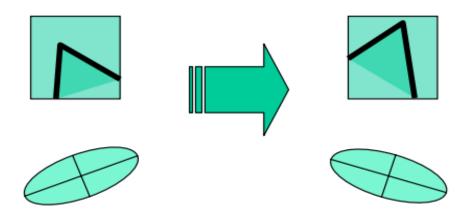
### Detected corner points



Credit: Markus Vincze, Technische Universität Wien

#### **Properties**

- Rotation invariant: corner response is invariant to image rotation
- Ellipse rotates with the corner but the shape and size (i.e. the eigenvalues) remain unchanged

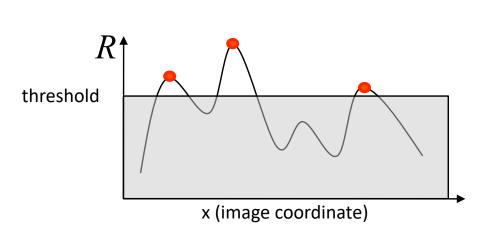


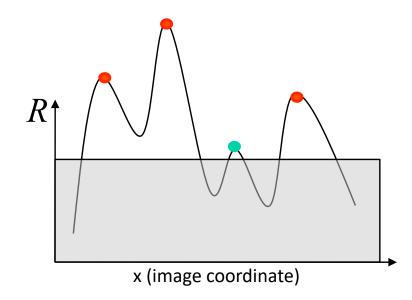
Credit: Markus Vincze, Technische Universität Wien

Partial invariance to affine intensity change

Only derivatives are used  $\Rightarrow$  invariance to intensity shift  $I \rightarrow I + b$ 

Intensity scale:  $I \rightarrow a I$ 





Credit: Kris Kitani, Carnegie Mellon University

The Harris	corner de	tector no	t invarian	t to char	nges in

### Demos

The following code is taken from the harrisCornerDetection project in the lectures directory of the ACV repository

#### See:

harrisCornerDetection.h harrisCornerDetectionImplementation.cpp harrisCornerDetectionApplication.cpp

```
Example use of openCV to find interest point features using the Harris corner detector
  Implementation file
      http://docs.opencv.org/2.4/modules/imgproc/doc/feature_detection.html?highlight=cornerharris#cornerharris
      http://docs.opencv.org/2.4/modules/features2d/doc/common interfaces of feature detectors.html#goodfeaturestotrackdetector
  David Vernon
  24 June 2017
*/
#include "harrisCornerDetection.h"
 * function harrisCornerDetection
 * Trackbar callback - set maximum number of corners / interest points
 * Trackbar callback - set minimum distance between matched points
 * Trackbar callback - set block size over which to compute gradients
*/
void harrisCornerDetection(int, void*) {
   extern Mat src;
   extern int num_corners;
   extern int quality level;
   extern int    min distance;
   extern int block size;
   extern char* feature_location_window_name;
   extern char* feature magnitude window name;
```

```
Mat cornerness image; // image of the response to the Harris corner detector
Mat corner image;
                     // image showing the selected Harris interest points
Mat image grey;
vector<KeyPoint> keypoints;
cvtColor(src, image grey, CV BGR2GRAY); // Harris operates on grey-scale images
if (num corners < 1) num corners = 1;// can't define trackbar lower limit so enforce it here</pre>
if (block size < 2) block size = 2; // can't define trackbar lower limit so enforce it here</pre>
if (min distance < 1) min distance = 2; // can't define trackbar lower limit so enforce it here</pre>
/* see http://docs.opencv.org/2.4/modules/imgproc/doc/feature detection.html?highlight=cornerharris#cornerharris */
cornerHarris(image grey,
                          // input
            cornerness_image, // output
                             // blocksize, i.e. the size of region over which to compute the autocorrelation matrix
            block size,
                              // the neighbourhood over which the partial derivatives are computed (using the Sobel operator)
            3,
                          // the Trace multiplier used in the Harris forumula
            0.04);
/* see http://docs.opencv.org/2.4/modules/features2d/doc/common interfaces of feature detectors.html#goodfeaturestotrackdetector */
                                                                      // maximum number of corners to detect
GoodFeaturesToTrackDetector harris detector(num corners,
                                                                     // quality level
                                           0.01,
                                                                     // minimum distance between matched points
                                           min distance,
                                                                     // block size over which to compute the autocorrelation matrix
                                           block size,
                                           true,
                                                                     // true to use Harris; false to use minimum eigenvalue
                                                                     // the Trace multiplier used in the Harris forumula
                                           0.04);
harris detector.detect(image grey,keypoints);
drawKeypoints(src, keypoints, corner image, Scalar( 0, 0, 255 ) );
Mat cornerness display image = convert 32bit image for display(cornerness image);
imshow(feature_magnitude_window_name, cornerness_display_image);
imshow(feature location window name, corner image);
```

# Reading

R. Szeliski, Computer Vision: Algorithms and Applications, Springer, 2010.

Section 4.1 Points and Patches

Section 4.1.1 Feature Detectors