Algorithms and Data Structures
CS-CO-412

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Trees

Lecture 8

Topic Overview

- Types of trees
- Binary Tree ADT
- **Binary Search Tree**
- Optimal Code Trees
- Huffman's Algorithm
- Height Balanced Trees
  - AVL Trees
  - Red-Black Trees

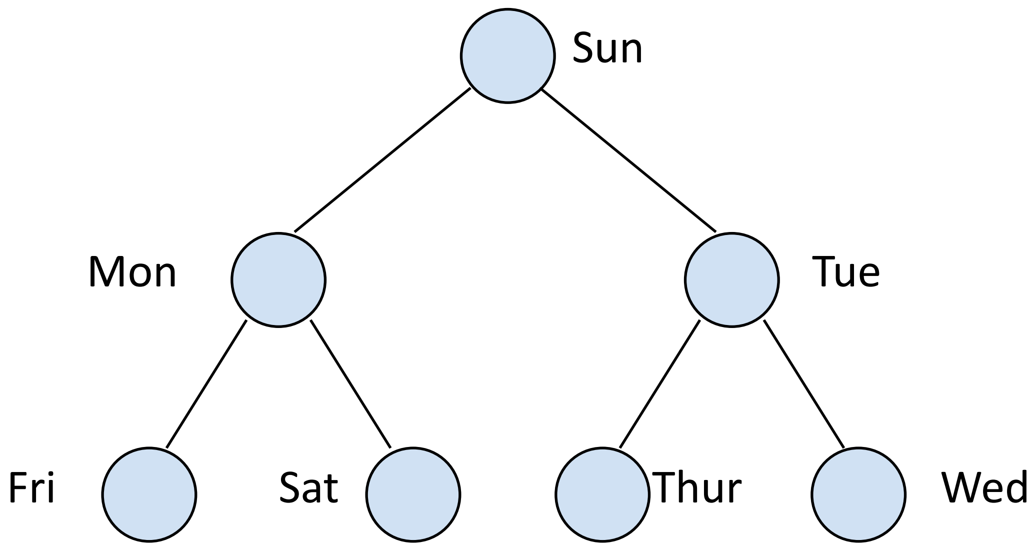
Binary Search Trees

- A Binary Search Tree (BST) is a special type of binary tree
  - it represents information is an ordered format
  - A binary tree is binary search tree if for every node *w*, all keys in the left subtree of i have values less than the key of *w* and all keys in the right subtree have values greater than key of *w*.

Binary Search Trees

- Definition: A binary search tree *T* is a binary tree; either it is empty or each node in the tree contains an identifier and:
  - all keys in the left subtree of *T* are less (numerically or alphabetically) than the identifier in the root node *T*;
  - all identifers in the right subtree of *T* are greater than the identifier in the root node *T*;
  - The left and right subtrees of *T* are also binary search trees.

Binary Search Trees



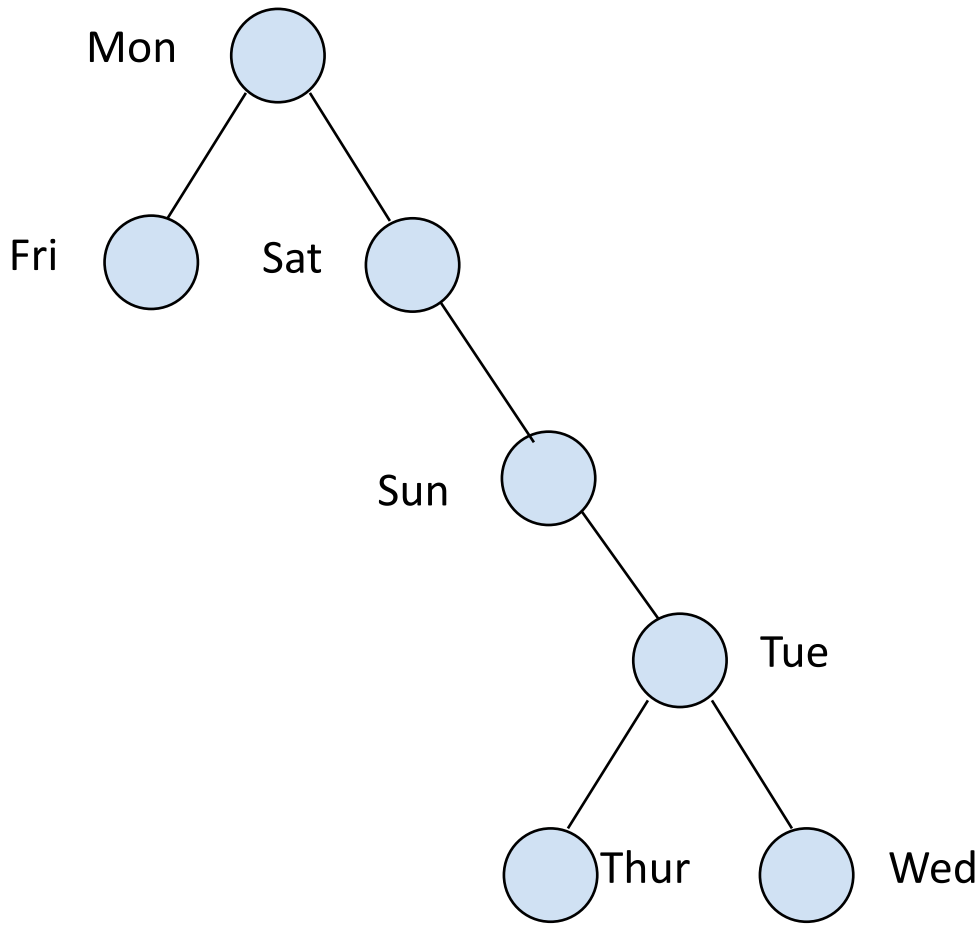
Binary Search Trees

- The main point to notice about such a tree is that, if traversed inorder, the keys of the tree (*i.e.* its data elements) will be encountered in a sorted fashion
-
- Furthermore, efficient searching is possible using the binary search technique
  - search time is $O(log_2n)$

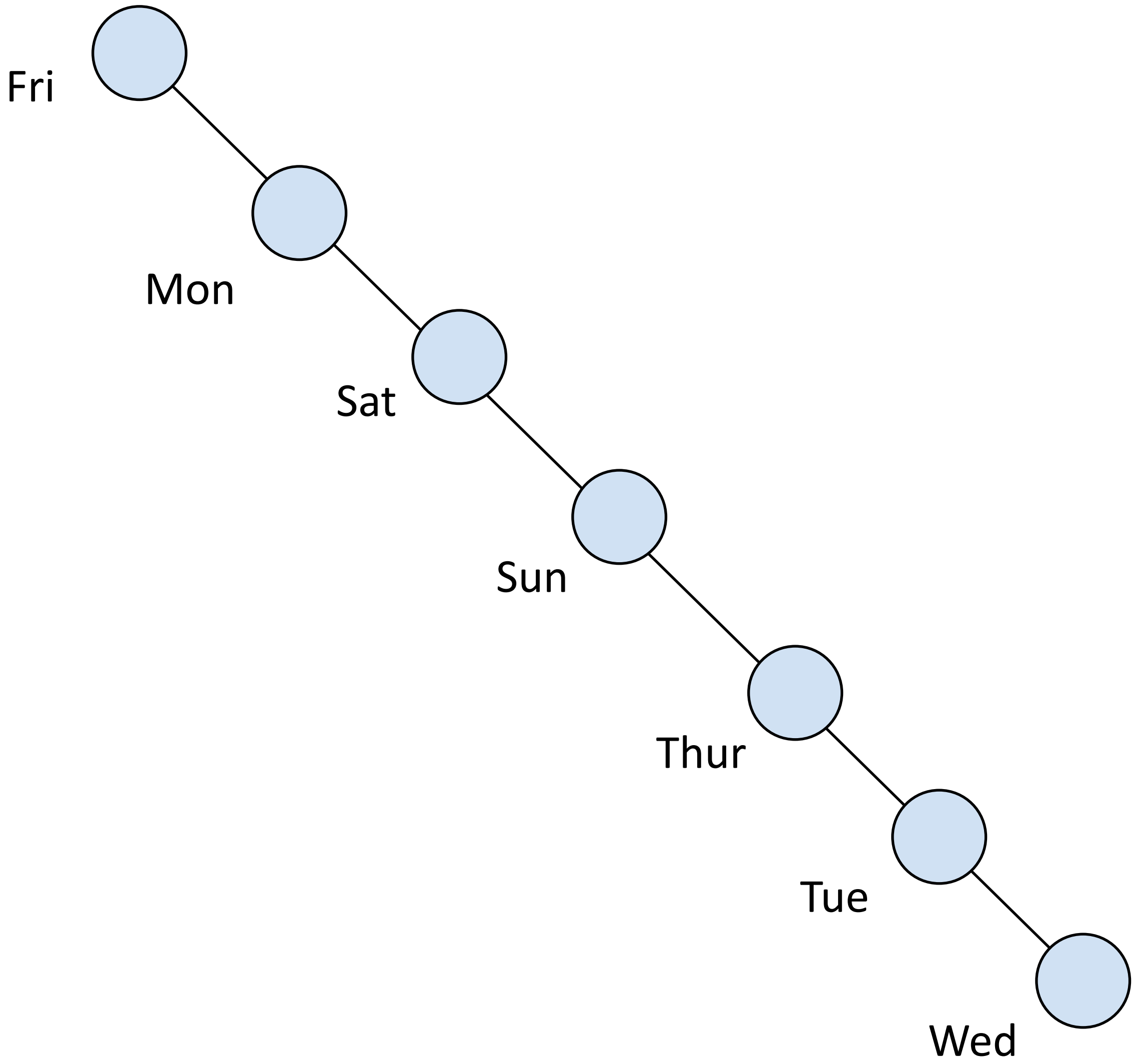
Binary Search Trees

- It should be noted that several binary search trees are possible for a given data set, *e.g,* consider the following tree:

Binary Search Trees



Binary Search Trees



Binary Search Trees

- Let us consider how such a situation might arise.

  To do so, we need to address how a binary search tree is constructed

  – Assume we are building a binary search tree of words
  –
  – Initially, the tree is null, i.e. there are no nodes in the tree
  –
  – The first word is inserted as a node in the tree as the root, with no children

Binary Search Trees

- On insertion of the second word, we check to see if it is the same as the key in the root, less than it, or greater than it
  - If it is the same, no further action is required (duplicates are not allowed)
  - If it is less than the key in the current node, move to the left subtree and *compare again*
  - If the left subtree does not exist, then the word does not exist and it is inserted as a new node on the left

Binary Search Trees

  - If, on the other hand, the word was greater than the key in the current node, move to the right subtree and compare again

  - If the right subtree does not exist, then the word does not exist and it is inserted as a new node on the right

– This insertion can most easily be effected in a recursive manner

Binary Search Trees

– The point here is that the structure of the tree depends on the order in which the data is inserted in the list

– If the words are entered in sorted order, then the tree will degenerate to a 1-D list

BST Operations

- *Insert*: E × BST → BST :

  The function value *Insert*(*e*,*T*) is the BST *T* with the element e inserted as a leaf node; if the element already exists, no action is taken.

BST Operations

- *Delete*: $E \times BST \rightarrow BST$:

  The function value *Delete*(*e*,*T*) is the BST *T* with the element e deleted; if the element is not in the BST exists, no action is taken.

Implementation of *Insert*(*e, T*)

- If *T* is empty (i.e. *T* is NULL)
  - create a new node for e
  - make *T* point to it
- If *T* is not empty
  - if *e* < element at root of *T*
    - Insert *e* in left child of *T*: *Insert(e, T(1))*
  - if *e* > element at root of *T*
    - Insert *e* in right child of *T: Insert(e, T(2))*

Implementation of *Delete*(*e, T*)

- First, we must locate the element e to be deleted in the tree
  - if e is at a leaf node
    - we can delete that node and be done
  - if e is at an interior node at w
    - we can't simply delete the node at w as that would disconnect its children
  - if the node at w has only one child
    - we can replace that node with its child

Implementation of *Delete*(*e, T*)

– if the node at *w* has two children

  - we replace the node at *w* with the lowest-valued element among the descendents of its right child

  - this is the left-most node of the right tree

  - It is useful to have a function DeleteMin with removes the smallest element from a non-empty tree and returns the value of the element removed

Implementation of *Delete*(*e, T*)

- If *T* is not empty
  - if *e* < element at root of *T*
    - Delete *e* from left child of *T*: *Delete(e, T(1))*
  - if *e* > element at root of *T*
    - Delete *e* from right child of *T: Delete(e, T(2))*
  - if *e* = element at root of *T* and both children are empty
    - Remove *T*
  - if *e* = element at root of *T* and left child is empty
    - Replace *T* with *T(2)*

Implementation of *Delete*(*e, T*)

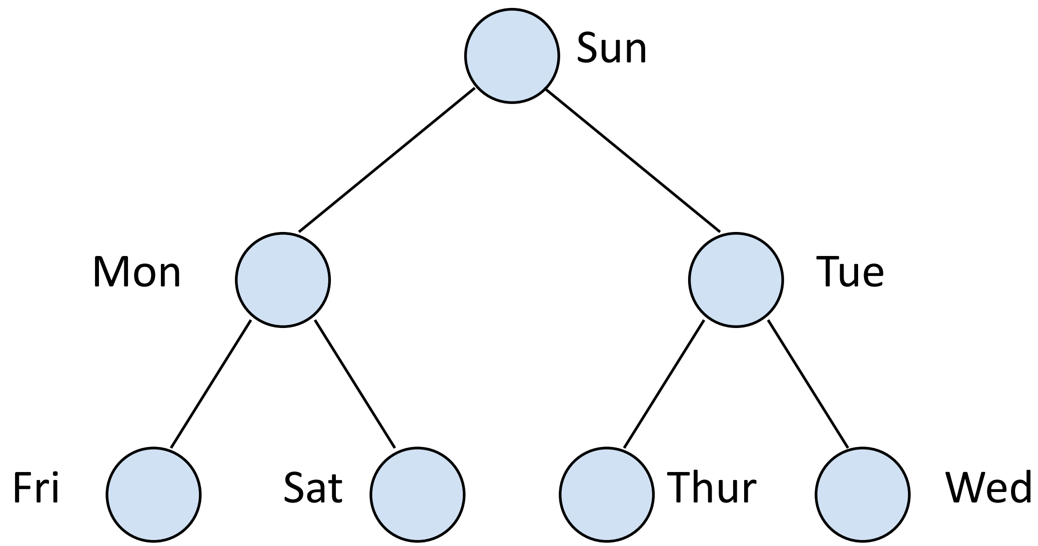
– if *e* = element at root of *T* and right child is empty

  - Replace *T* with *T(1)*

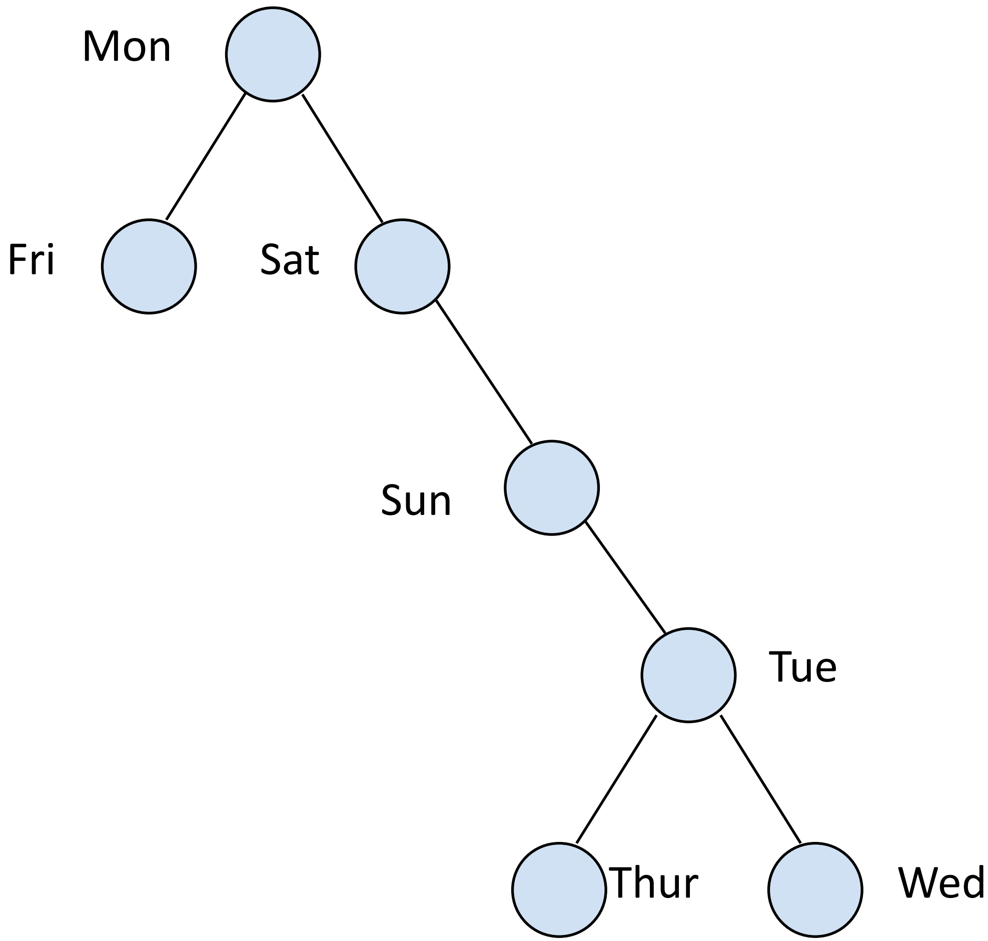
– if *e* = element at root of *T* and neither child is empty

  - Replace *T* with left-most node of *T(2)*

Implementation of *Delete*(*e*, *T*)



Implementation of *Delete*(*e, T*)



BST Implementation

```
/* implementation of BST ADT */

#include <stdio.h>
#include <math.h>
#include <string.h>

#define FALSE 0
#define TRUE 1

typedef struct {
           int number;
           char *string;
        } ELEMENT_TYPE;
```


BST Implementation

```
typedef struct node *NODE_TYPE;

typedef struct node{
           ELEMENT_TYPE element;
           NODE_TYPE left, right;
        } NODE;

typedef NODE_TYPE BST_TYPE;
typedef NODE_TYPE WINDOW_TYPE;
```

BST Implementation

```
/*** insert an element in a BST ***/

BST_TYPE *insert(ELEMENT_TYPE e, BST_TYPE *tree) {
   WINDOW_TYPE temp;
   if (*tree == NULL) {
      /* we are at an external node: create a new node */
      /* and insert it                                 */
      if ((temp =(NODE_TYPE) malloc(sizeof(NODE))) == NULL)
         error(“insert: unable to allocate memory”);
      else {
         temp->element = e;
         temp->left = NULL;
         temp->right = NULL;
         *tree = temp;
      }
```


BST Implementation

```
    }
    else if (e.number < (*tree)->element.number) {
       /* assume number field is the key */
       insert(e, &((*tree)->left));
    }
    else if (e.number > (*tree)->element.number) {
       insert(e, &((*tree)->right));
    }

    /* if e.number == (*tree)->element.number, e is */
    /* already in the tree so do nothing            */

    return(tree);
}
```


BST Implementation

```
/*** return and delete the smallest node in a tree ***/
/*** i.e. return and delete the left-most node     ***/

ELEMENT_TYPE delete_min(BST_TYPE *tree) {
   ELEMENT_TYPE e;
   BST_TYPE p;
   if ((*tree)->left == NULL) {

      /* (*tree) points to the smallest element */

      e = (*tree)->element;

      /* replace the node pointed to by tree */
      /* by its right child                  */
```


BST Implementation

```
        p = *tree;
        *tree = (*tree)->right;
        free(p);

        return (e);
    }
    else {

        /* the node pointed to by *tree has a left child */

        return(delete_min(&((*tree)->left)));
    }
}
```

BST Implementation

```
/*** delete an element from a BST ***/

BST_TYPE *delete(ELEMENT_TYPE e, BST_TYPE *tree) {
   BST_TYPE p;;
   if (*tree != NULL) {
      if (e.number < (*tree)->element.number)
         delete(e, &((*tree)->left));
      else (e.number > (*tree)->element.number)
         delete(e, &((*tree)->right));
      else if (((*tree)->left == NULL) &&
               ((*tree)->right == NULL)) {

         /* leaf node containing e: delete it */
```


BST Implementation

```
      /* leaf node containing e: delete it */

      p = *tree;
      free(p);
      *tree = NULL;
   }
   else if ((*tree)->left == NULL) {
      /* internal node containing e and it has only */
      /* a right child; delete it and make tree     */
      /* point to the right child                   */

      p = *tree;
      *tree = (*tree)->right;
      free(p);
   }
```

BST Implementation

```
else if ((*tree)->right == NULL) {

   /* internal node containing e and it has only */
   /* a left child; delete it and make tree     */
   /* point to the left child                   */

   p = *tree;
   *tree = (*tree)->left;
   free(p);
}
```


BST Implementation

```
        else {
           /* internal node containing e and it has both */
           /* left and right children; replace it with   */
           /* the leftmost node of the right child       */

           (*tree)->element = delete_min(&((*tree)->right));

        }
     }
  }
```


BST Implementation

```
/*** inorder traversal of a tree,                        ***/
/*** printing node elements                              ***/
/*** parameter n is the current level in the tree        ***/

int inorder(BST_TYPE *tree, int n) {
   int i;
   if (*tree != NULL) {
      inorder(tree->left, n+1);
      for (i=0; i<n; i++) printf("      ");
      printf("%d %s\n",tree->element.number,
                       tree->element.string);
      inorder(tree->right, n+1);
   }
}
```

BST Implementation

```
/*** print all elements in a tree by traversing   ***/
/*** inorder                                      ***/

int print(BST_TYPE *tree) {

   printf("Contents of tree by inorder traversal: \n");

   inorder(tree, 0);

   printf("-----------");
}
```


BST Implementation

```
/*** error handler: print message passed as argument and
     take appropriate action                         ***/
int error(char *s); {
   printf("Error: %s\n", s);
   exit(0);
}

/*** assign values to an element ***/

int assign_element_values(ELEMENT_TYPE *e, int number, char s[])
  {
   e->string = (char *) malloc(sizeof(char) * (strlen(s)+1));
   strcpy(e->string, s);
   e->number = number;
}
```


BST Implementation

```
/*** main driver routine ***/

   ELEMENT_TYPE e;
   BST_TYPE tree;
   int i;

   print(tree);

   assign_element_values(&e, 3, "...");
   insert(e, &tree);
   print(tree);

   assign_element_values(&e, 1, "+++");
   insert(e, &tree);
   print(tree);
```

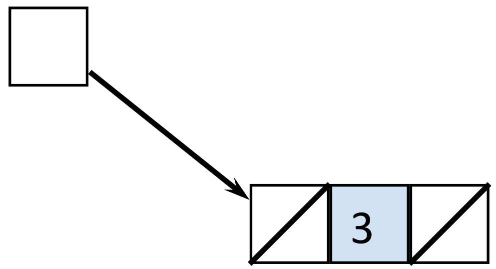
BST Implementation

```
assign_element_values(&e, 5, "---");
insert(e, &tree);
print(tree);

assign_element_values(&e, 2, ",,,");
insert(e, &tree);
print(tree);

assign_element_values(&e, 4, "***");
insert(e, &tree);
print(tree);

assign_element_values(&e, 6, "000");
insert(e, &tree);
print(tree);
```

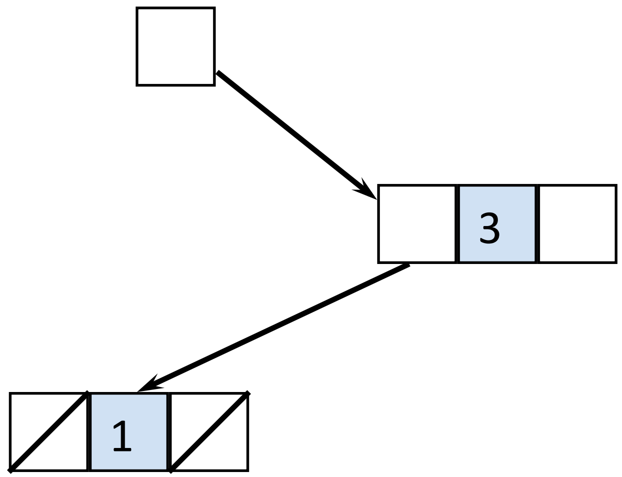

BST Implementation

```
    assign_element_values(&e, 3, "...");
    insert(e, &tree);
    print(tree);
}
```

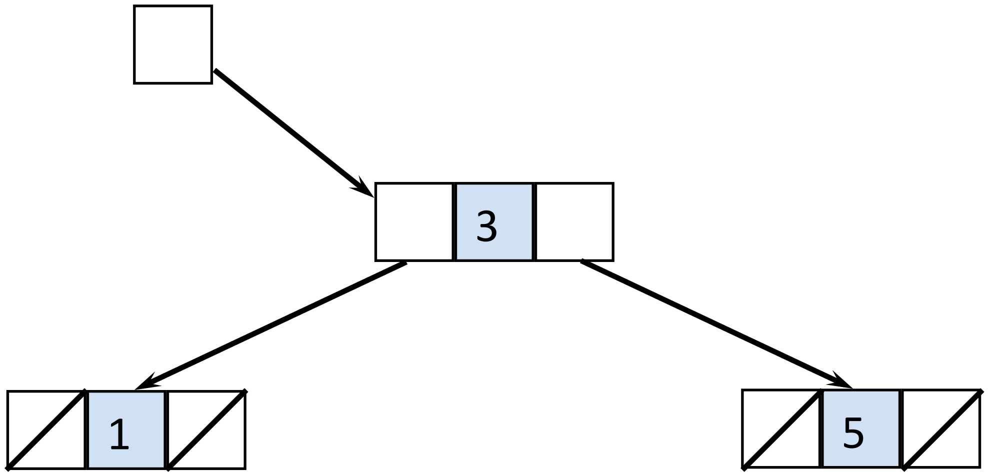

BINARY_TREE Implementation



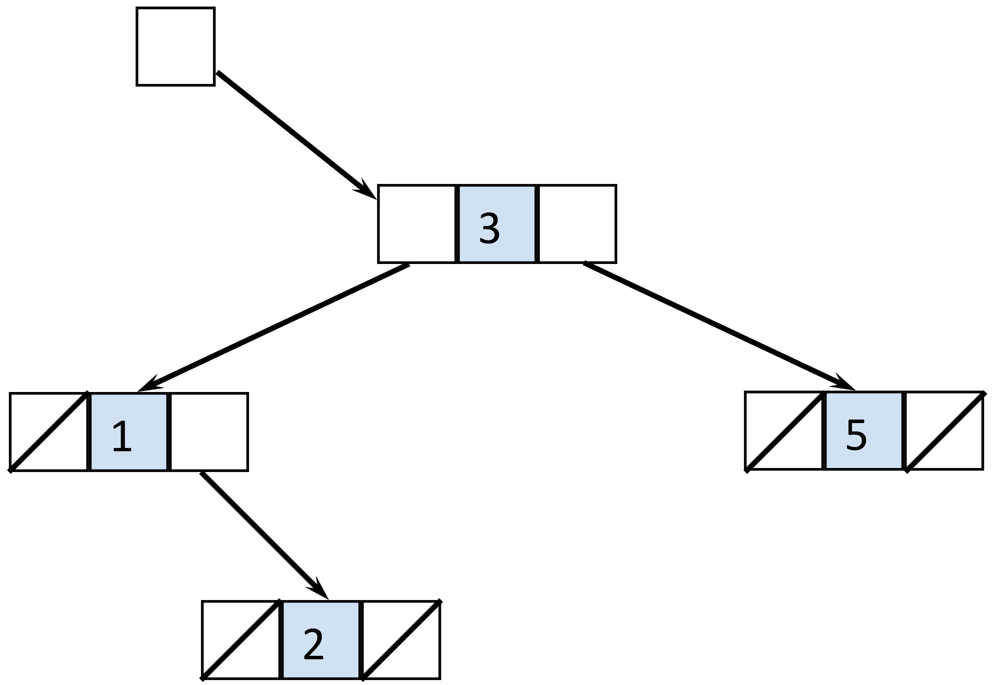
BINARY_TREE Implementation



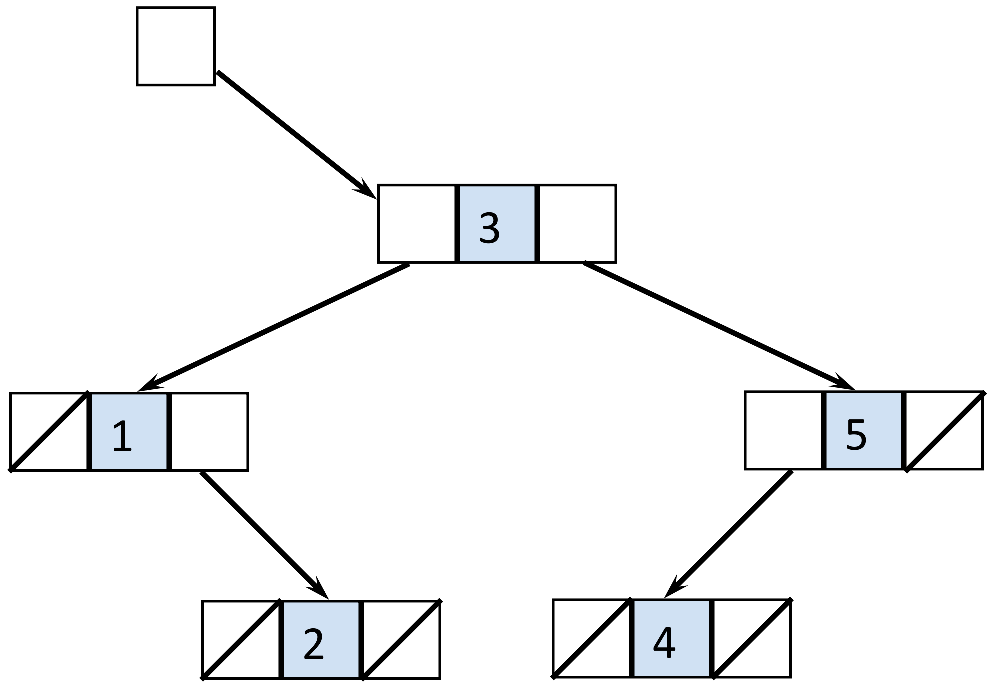
BINARY_TREE Implementation



BINARY_TREE Implementation



BINARY_TREE Implementation



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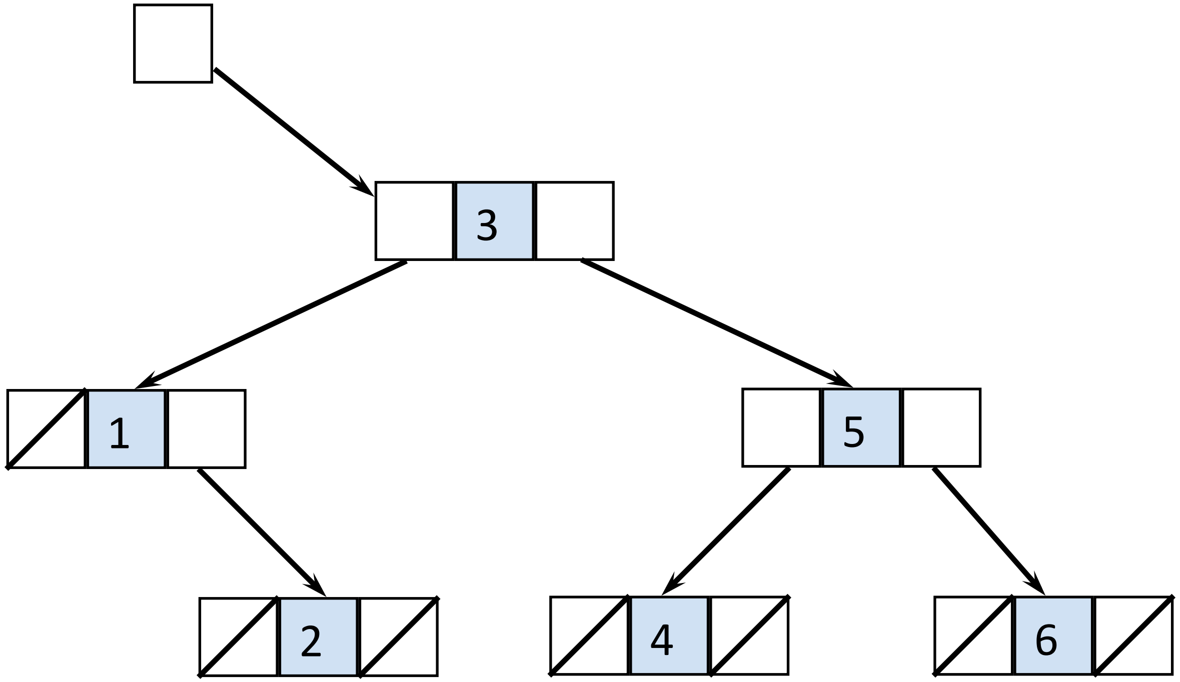
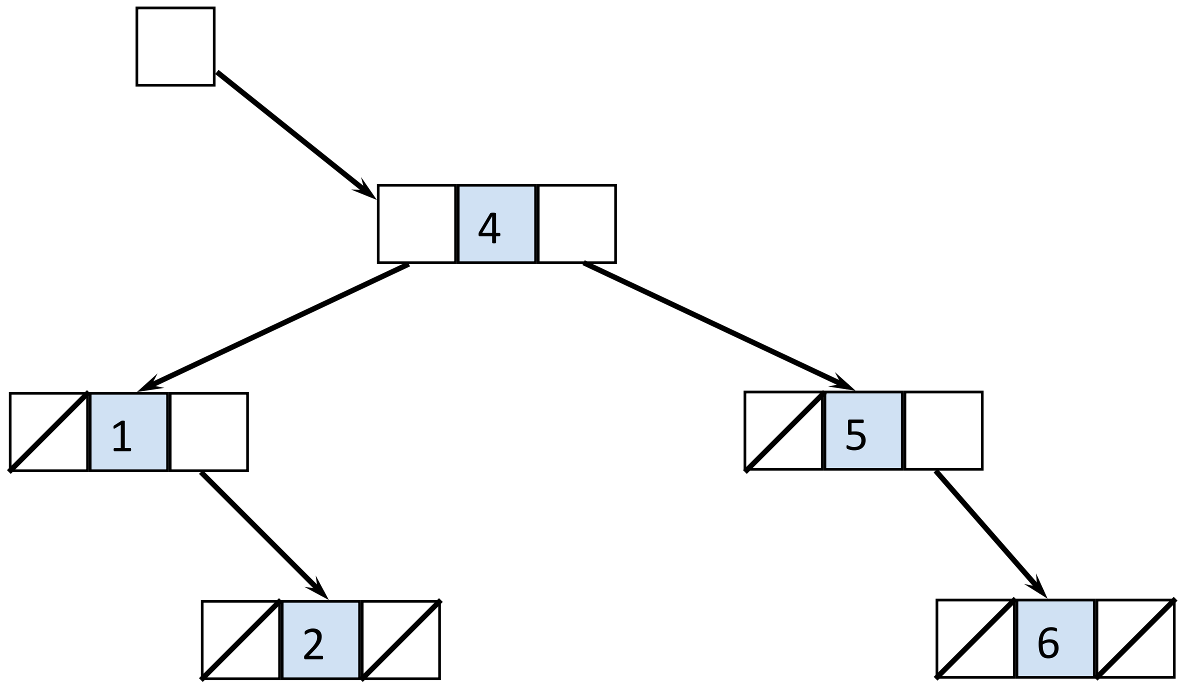


BINARY_TREE Implementation



Tree Traversals

- To perform a traversal of a data structure, we use a method of visiting every node in some predetermined order

- Traversals can be used

  – to test data structures for equality
  – to display a data structure
  – to construct a data structure of a give size
  – to copy a data structure

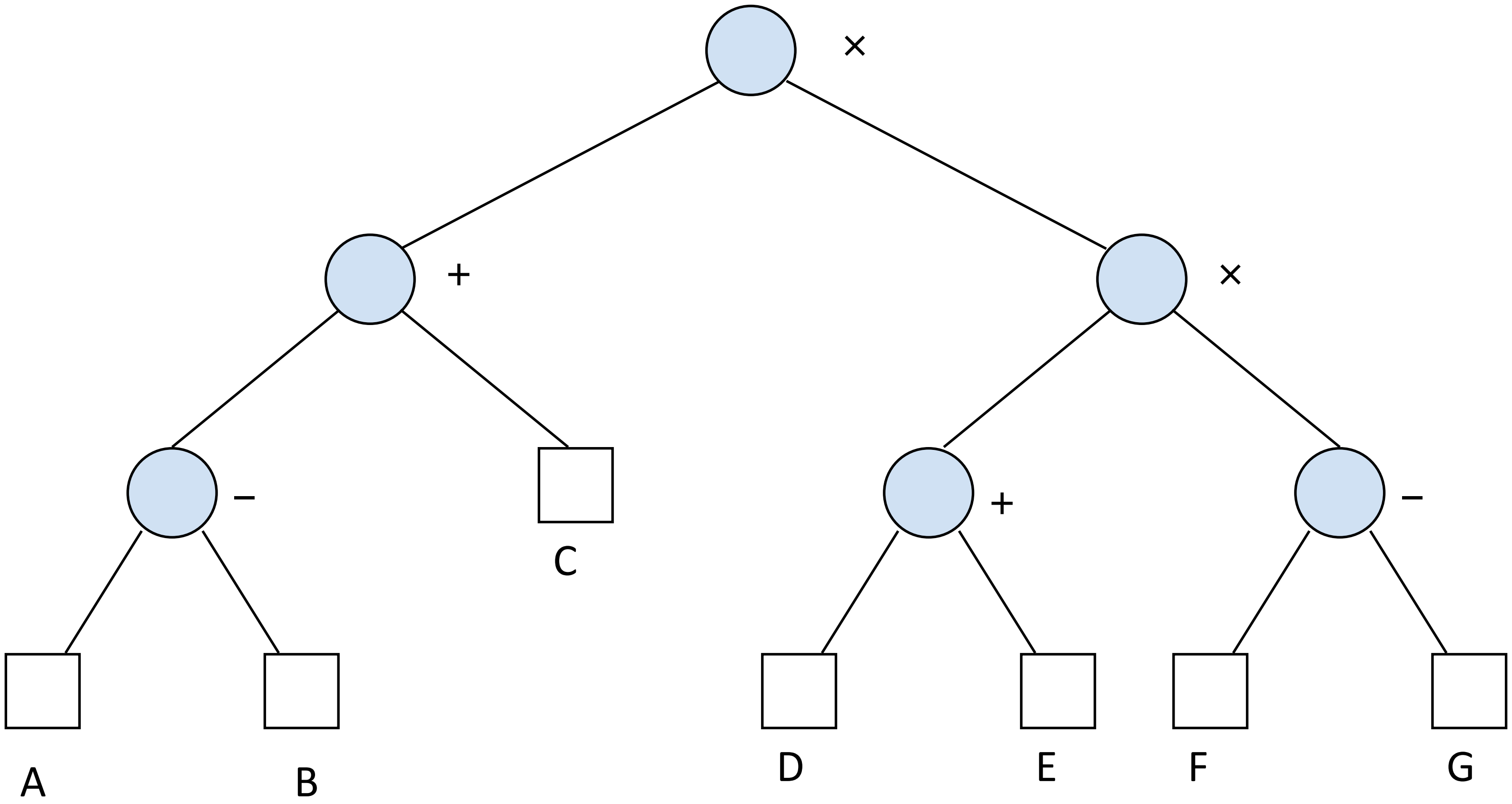
Depth-First Traversals

- There are 3 depth-first traversals

  – Inorder
  – Postorder
  – Preorder

- For example, consider the expression tree:

Example: Expression Tree



Depth-First Traversals

- Inorder traversal

  $A - B + C \times D + E \times F - G$

- Postorder traversal

  $A\ B - C + D\ E + F\ G - \times \times$

- Preorder traversal

  $\times + - A\ B\ C \times + D\ E - F\ G$

Depth-First Traversals

- The parenthesised Inorder traversal

  $((A - B) + C) \times ((D + E) \times (F - G))$

  This is the infix expression corresponding to the expression tree

- Postorder traversal gives a postfix expression

- Preorder traversal gives a prefix expression

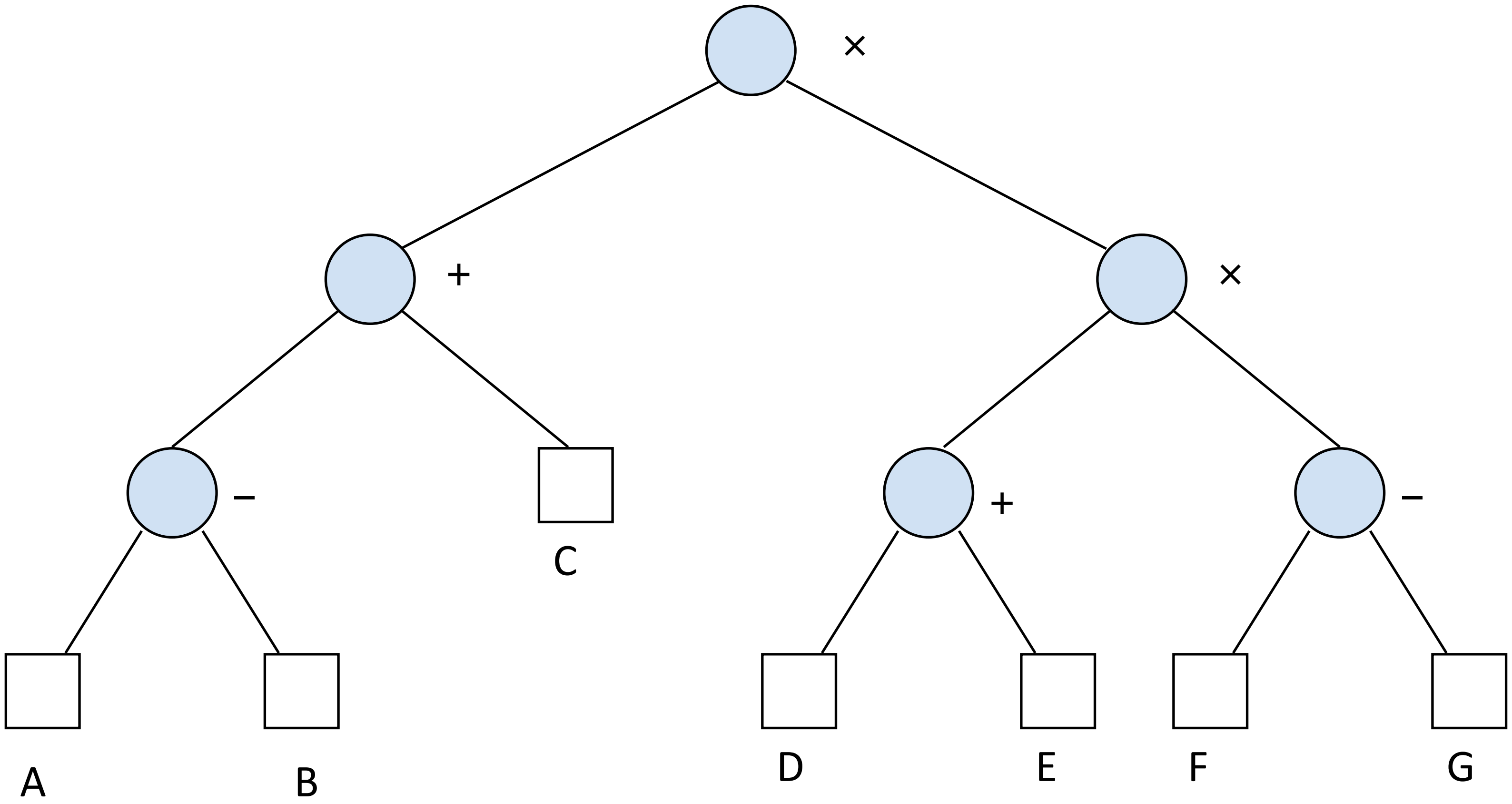
Depth-First Traversals

- Recursive definition of inorder traversal

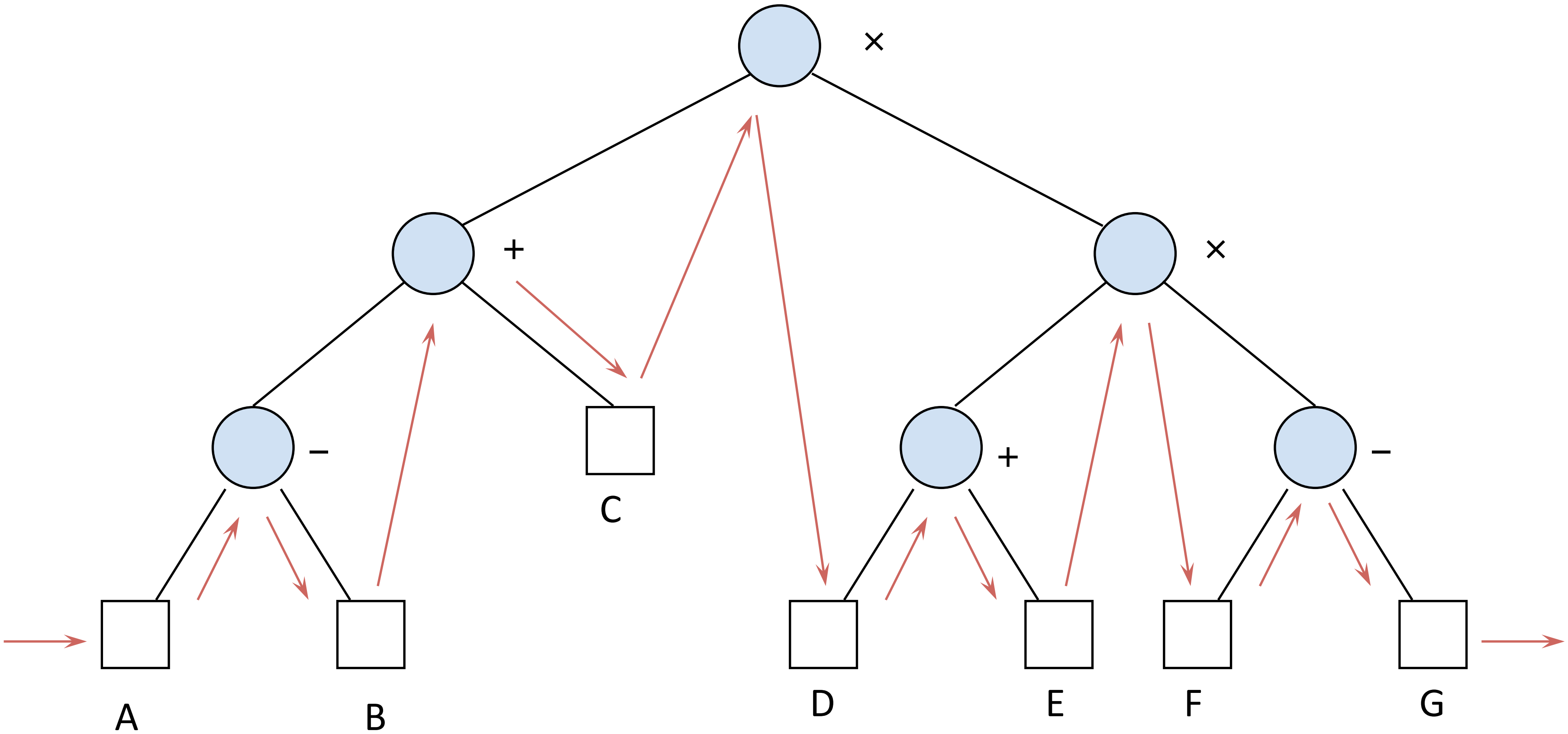
  Given a binary tree *T*

  if *T* is empty
    visit the external node
  otherwise
    perform an inorder traversal of *Left*(*T*)
    visit the root of *T*
    perform an inorder traversal of *Right*(*T*)

Example: Inorder Traversal



Example: Inorder Traversal



Depth-First Traversals

- Recursive definition of postorder traversal

  Given a binary tree *T*

  if *T* is empty

  visit the external node

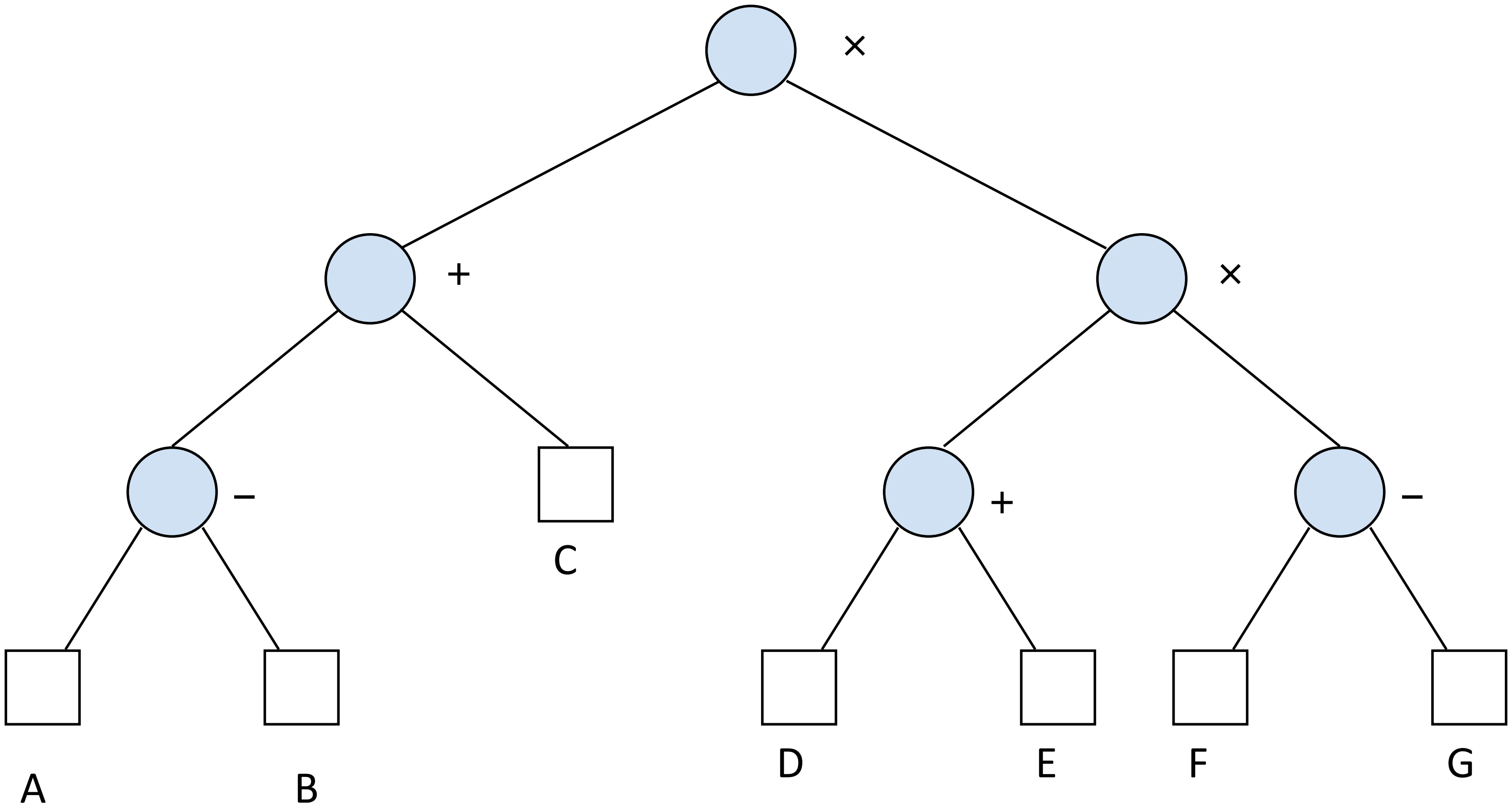
  otherwise

  perform an postorder traversal of *Left*(*T*)

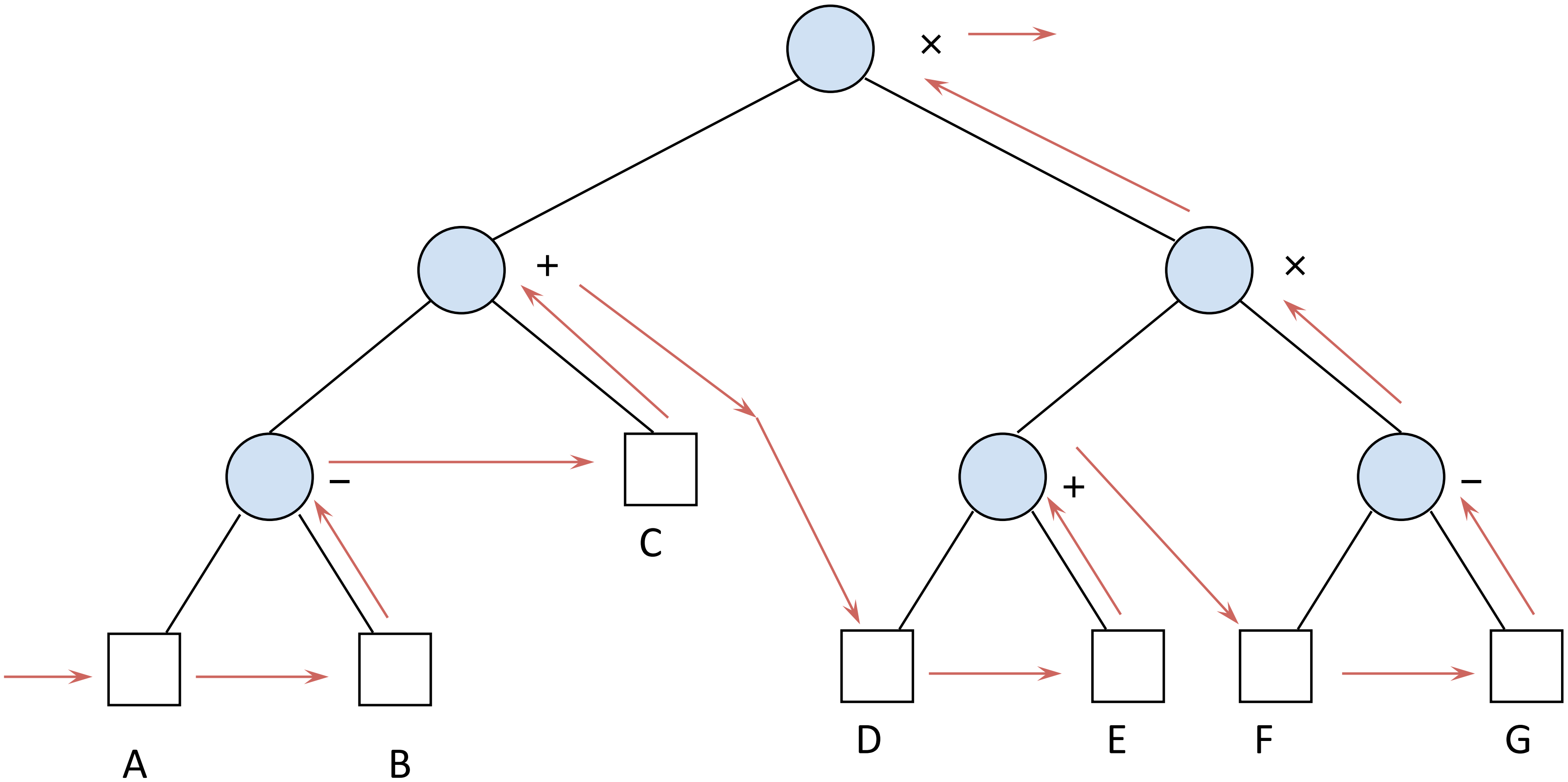
  perform an postorder traversal of *Right*(*T*)

  visit the root of *T*

Example: Postorder Traversal



Example: Postorder Traversal



Depth-First Traversals

- Recursive definition of preorder traversal

  Given a binary tree *T*

  if *T* is empty

  visit the external node

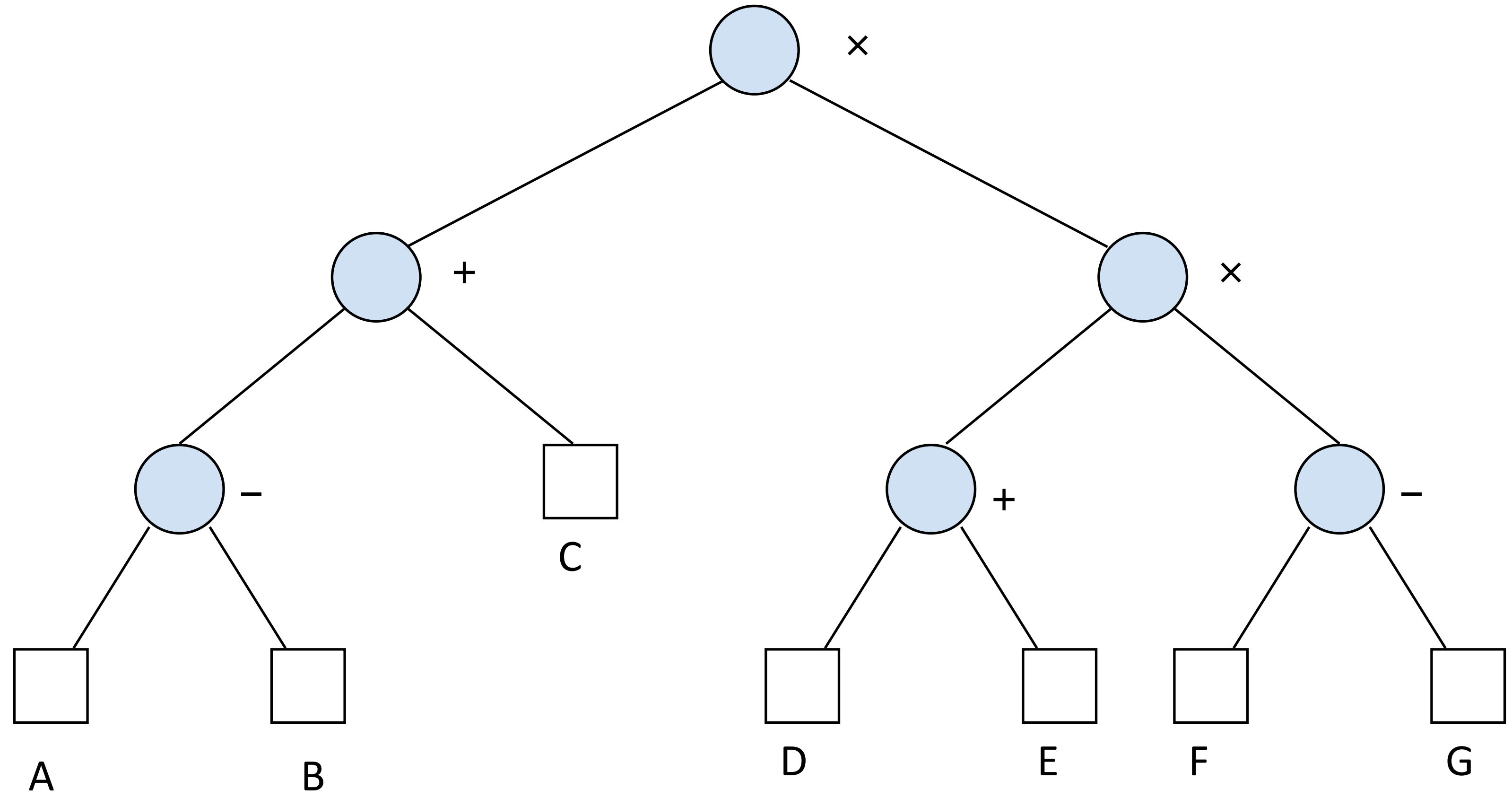
  otherwise

  visit the root of *T*

  perform an preorder traversal of *Left*(*T*)

  perform an preorder traversal of *Right*(*T*)

Example: Preorder Traversal



Example: Preorder Traversal