# Algorithms and Data Structures
## CS-CO-412

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# Graphs

Lecture 13

# Topic Overview

- Graphs
  - Types of graph
  - Adjacency matrix representation
  - Adjacency list representation
  - Breadth-First Search traversal
  - Depth-First Search traversal
  - Topological Sorting
  - **Minimum Spanning Tree**
    - **Prim's Algorithm**
    - **Kruskal's algorithm**
  - Shortest Path Algorithms
    - Dijkstra's algorithm
    - Floyd's algorithm

# Minimum Spanning Tree

- A *spanning tree* is a graph $G = (V, E)$ is a subset of edges from $E$ forming a tree connecting all vertices of $V$

- For edge-weighted graphs we are interested in the *minimum spanning tree:*

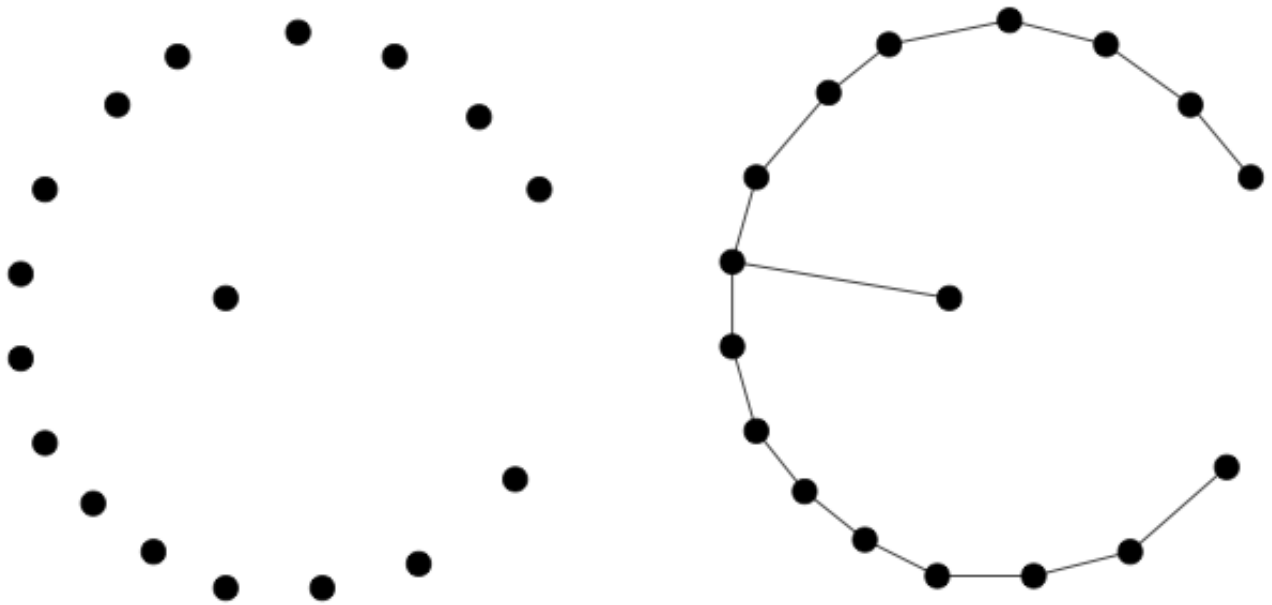
  – The tree whose sum of edge weights is as small as possible

# Minimum Spanning Tree

- Example application:
  - find the smallest amount of a connector (road, wire, pipe)

    required to connect a set of points (cities, houses, components)

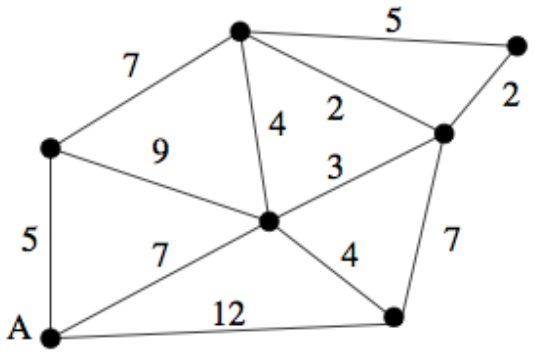
# Minimum Spanning Tree



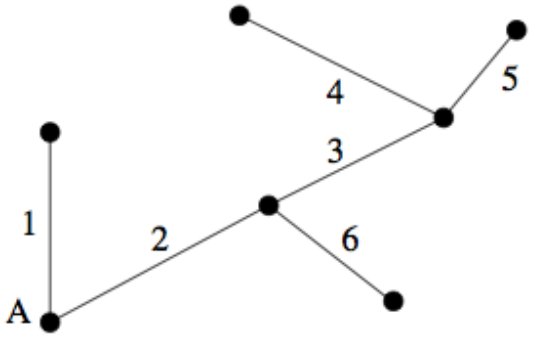
# Minimum Spanning Tree

- A minimum spanning tree minimizes the total length (weight) over all possible spanning trees
- There can be more than one  minimum spanning tree in a graph
- Two main algorithms
  - Prim's Algorithm
  - Kruskal's Algorithm

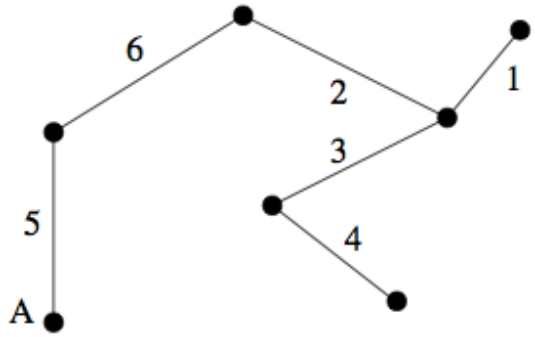
# Minimum Spanning Tree




G




Prim(G,A)




Kruskal(G)

Numbers denote order of insertion

# Minimum Spanning Tree

- Prim's Algorithm
  - Greedy algorithmic strategy:

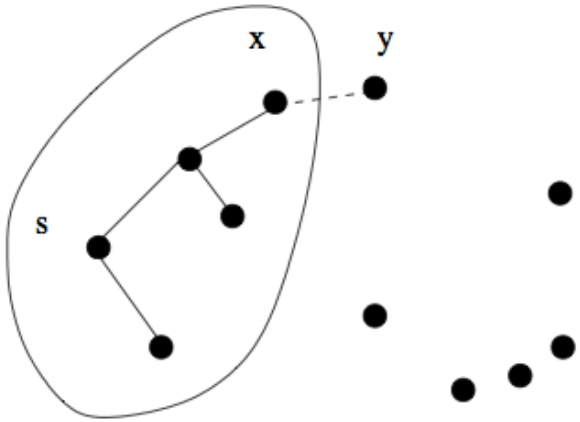
    Make a decision about what to do next by selecting the best *local* option from all available choices without regard to *global* structure,

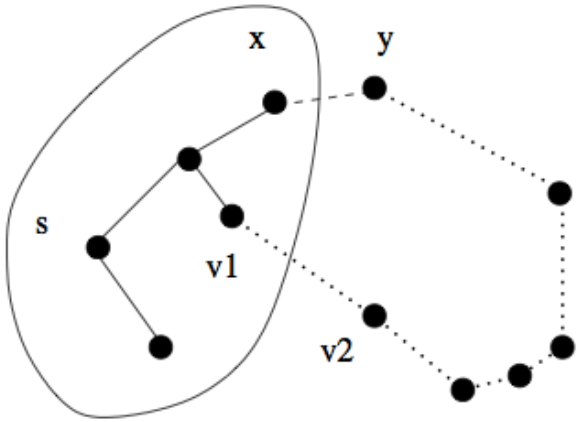
    do it,

    and then repeat until finished

# Minimum Spanning Tree

- Prim's Algorithm
  - Repeatedly select the smallest weight edge that will enlarge the number of vertices in the tree
    - Start from one (any) vertex,
    - Grow the rest of the tree, one edge at a time,
    - Until all vertices are included



# Minimum Spanning Tree

- Prim's Algorithm

  Prim-MST(G):

  Select an arbitrary vertex s to start the tree from

  While (there are still non-tree vertices remaining)
    Select the edge of minimum weight between a tree & non-tree vertex
    Add the selected edge and vertex to the tree T_prim

# Minimum Spanning Tree

```
/* Prim's algorithm                                                    */

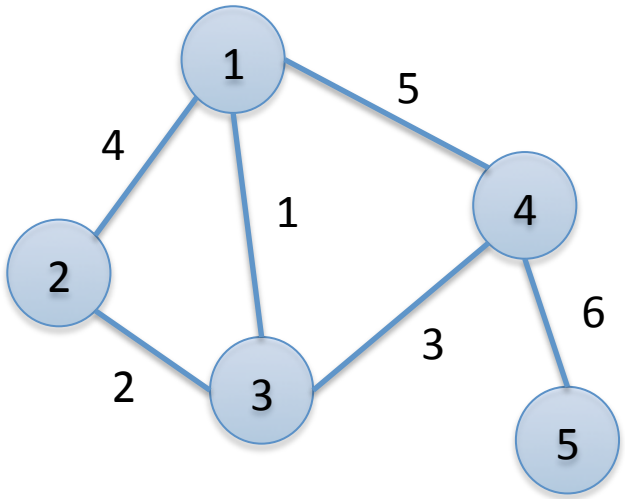
prim(graph *g, int start) {
   int i;                  /* counter                                 */
   edgenode *p;            /* temporary pointer                       */
   bool intree[MAXV+1];    /* is the vertex in the tree yet?          */
   int distance[MAXV+1];   /* cost of adding to tree                  */
   int parent[MAXV+1];     /* parent vertex                           */
   int v;                  /* current vertex to process               */
   int w;                  /* candidate next vertex                   */
   int weight;             /* edge weight                             */
   int dist;               /* best current distance from start        */

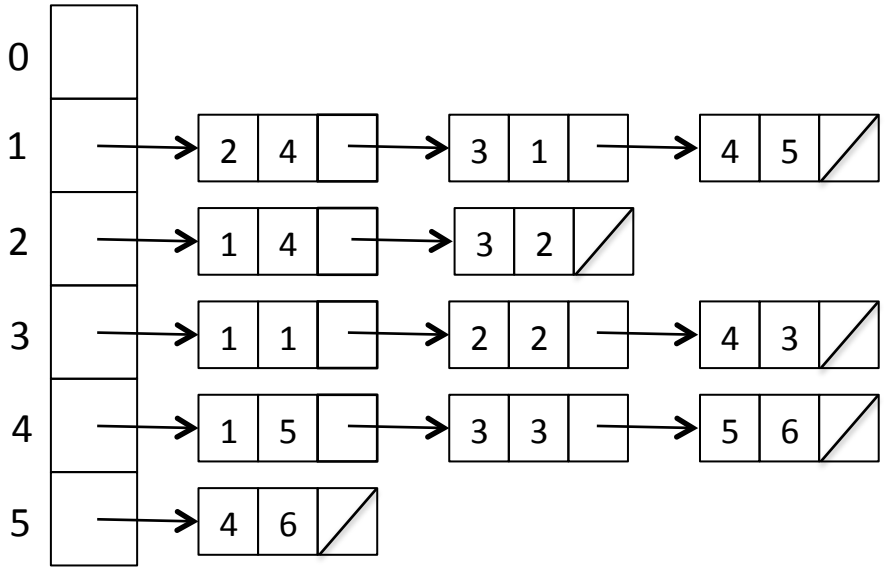
   for (i=1; i<=g->nvertices; i++) {
      intree[i] = FALSE;
      distance[i] = MAXINT;
      parent[i] = -1;
   }

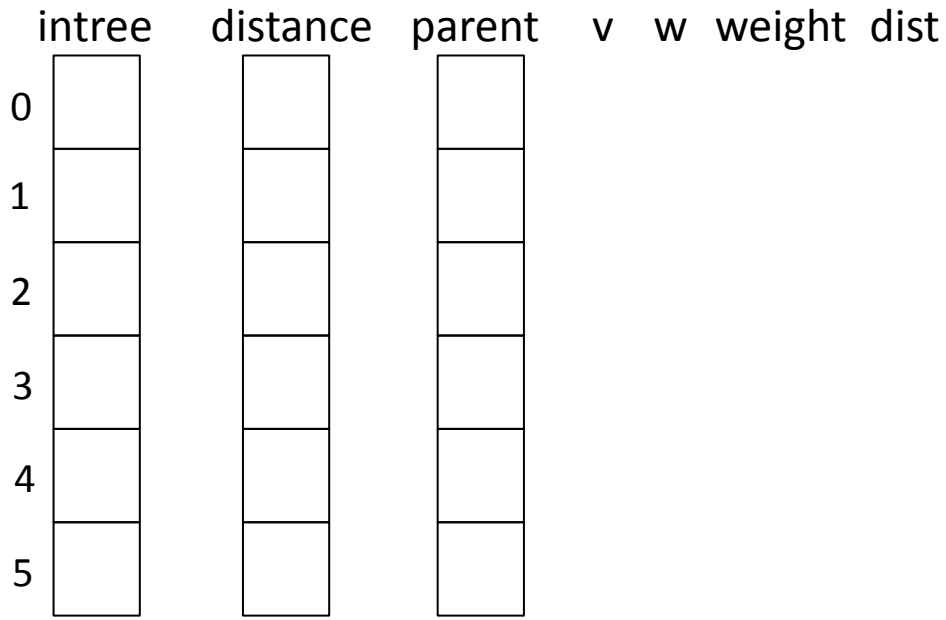
   distance[start] = 0;
   v = start;
```

# Minimum Spanning Tree

```
   while (intree[v] == FALSE) {
      intree[v] = TRUE;
      p = g->edges[v];
      while (p != NULL) {
         w = p->y;
         weight = p->weight;
         if ((weight < distance[w]) && (intree[w] == FALSE)) {
            distance[w] = weight;
            parent[w] = v;
         }
         p = p->next;
      }
      v = 1;
      dist = MAXINT;
      for (i=1; i<=g->nvertices; i++)
         if ((intree[i] == FALSE) && (distance[i] < dist)) {
            dist = distance[i];
            v = i;
         }
   }
}
```



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while (intree[v] == FALSE) {

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      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
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}
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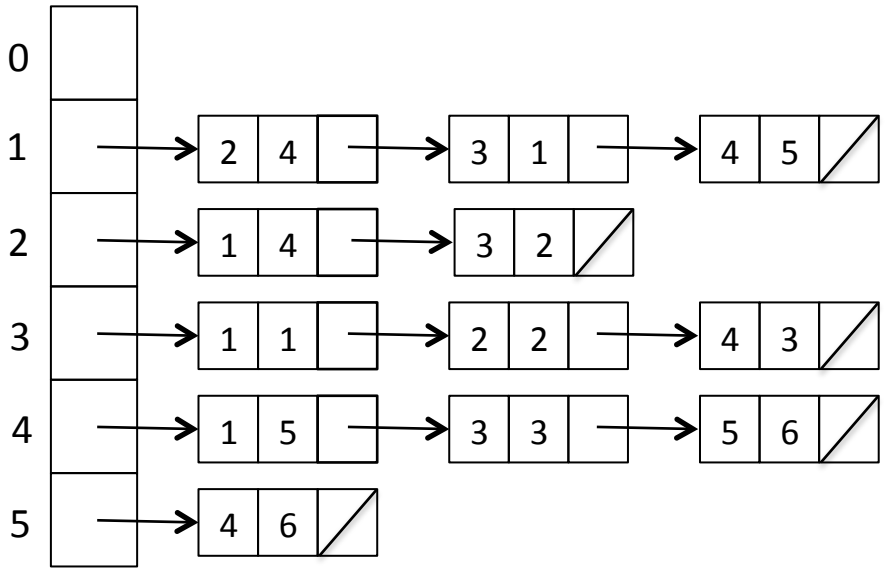


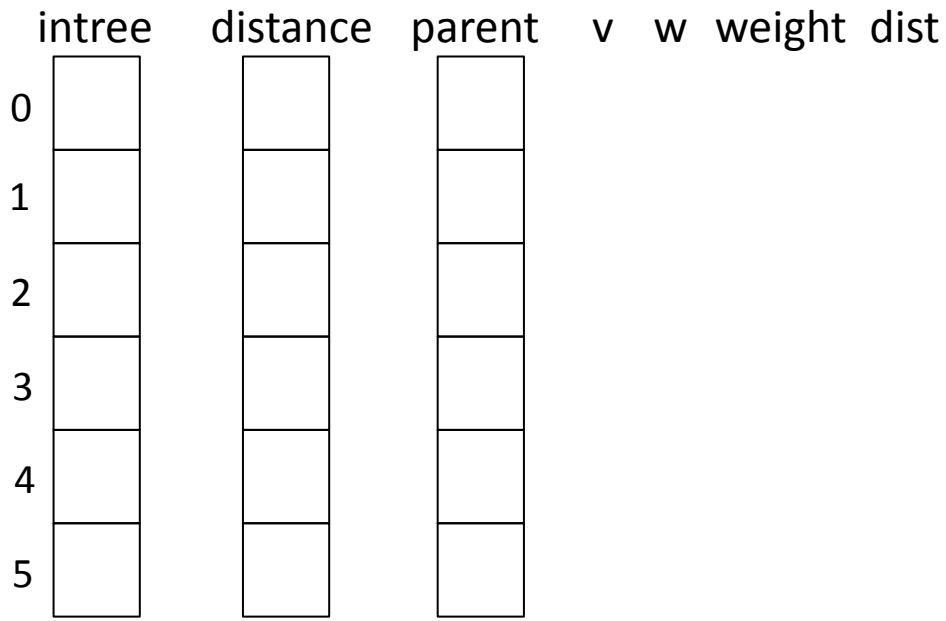

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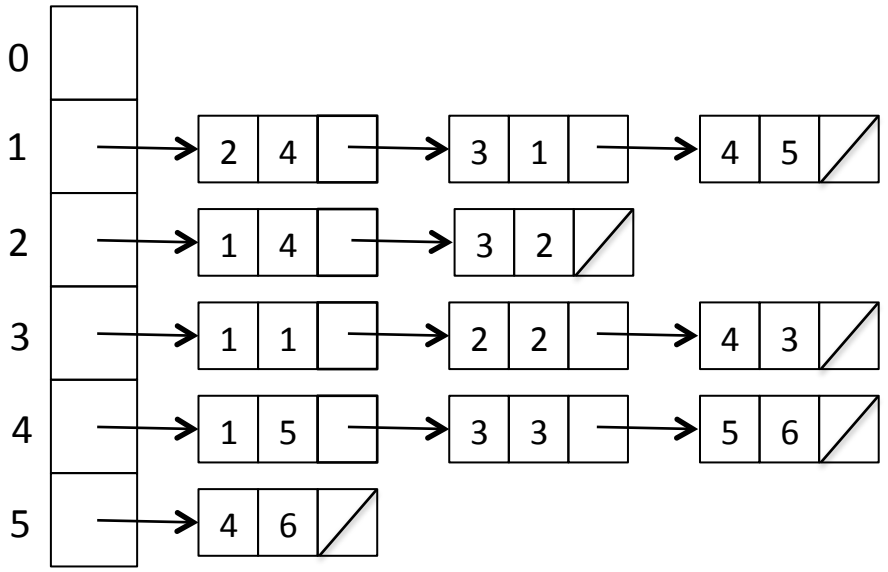


| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | F | ∞ | -1 |
| 2 | F | ∞ | -1 |
| 3 | F | ∞ | -1 |
| 4 | F | ∞ | -1 |
| 5 | F | ∞ | -1 |

v w weight dist

```
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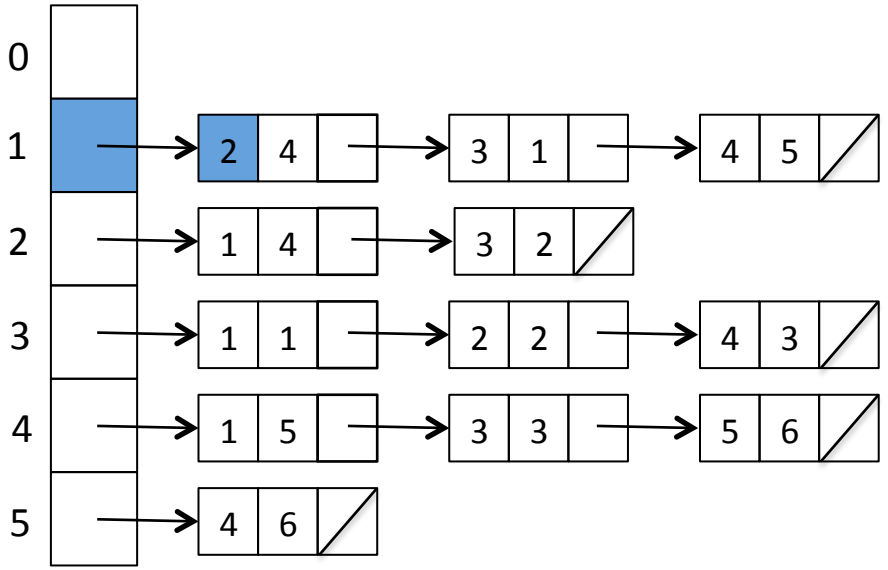


| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
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| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | |

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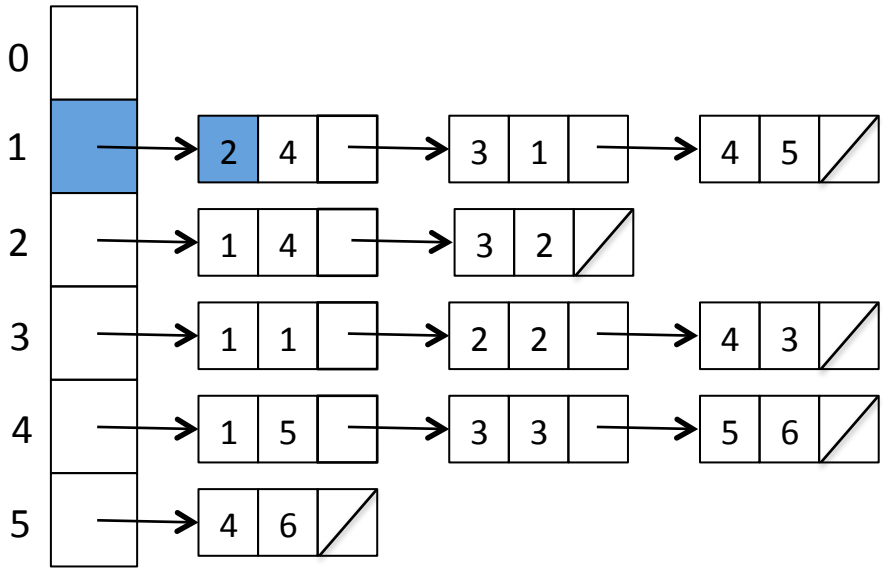


| | intree | distance | parent |
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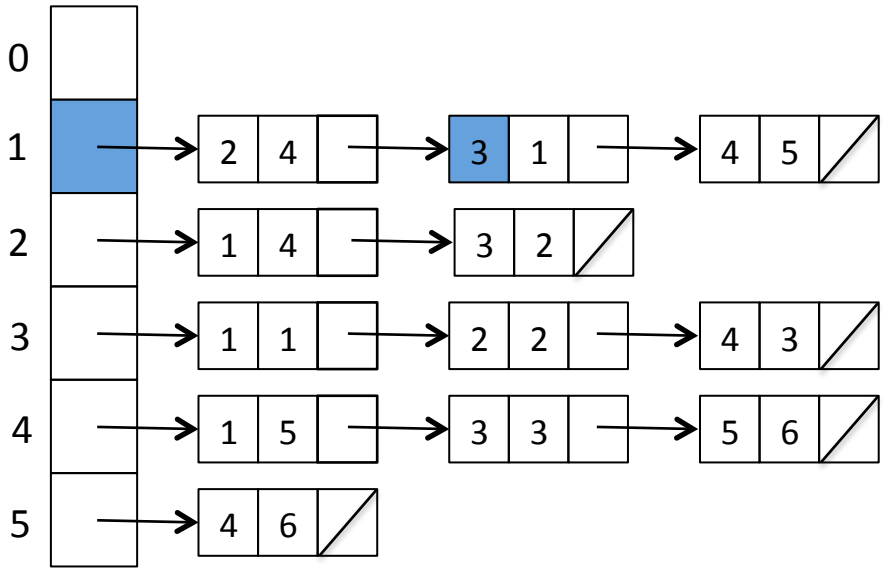


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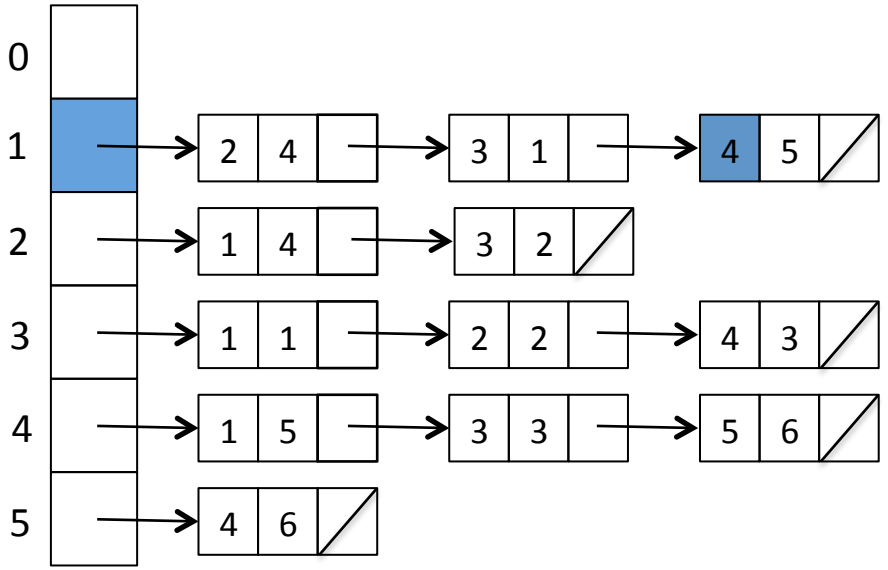


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| v | w | weight | dist |
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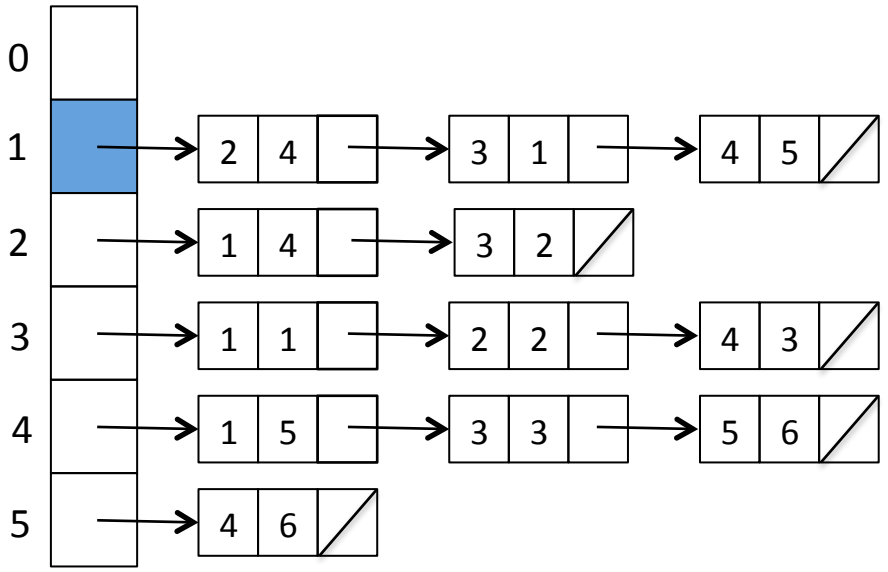


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| 0 | | | |
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| v | w | weight | dist |
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| 3 | | | |

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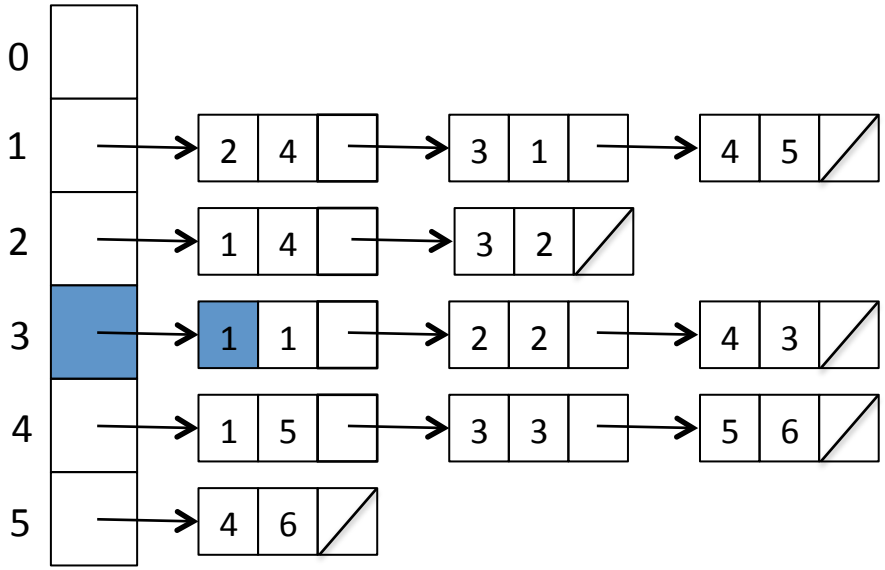


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| v | w | weight | dist |
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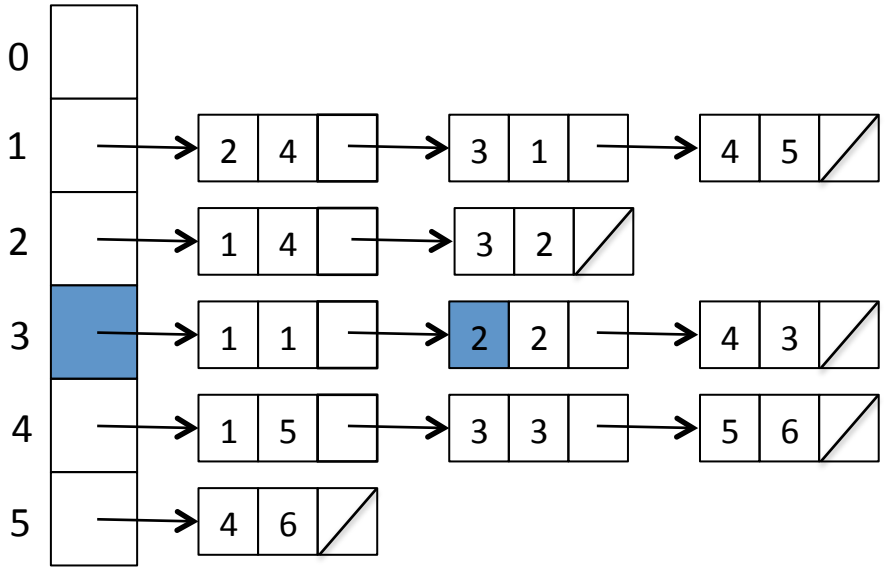


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| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | ∞ |
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| | 2 | 2 | |

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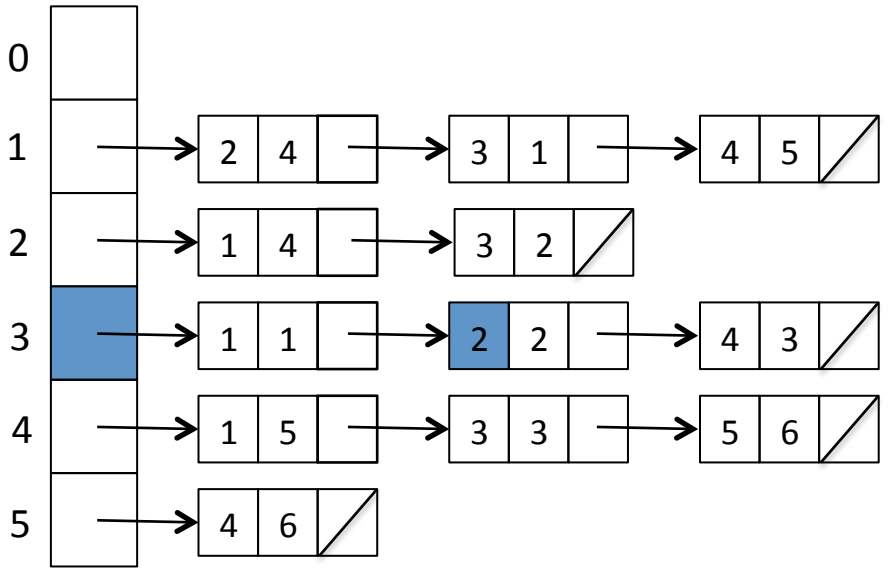


| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | F | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | F | 5 | 1 |
| 5 | F | ∞ | -1 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | ∞ |
| 1 | 3 | 1 | 4 |
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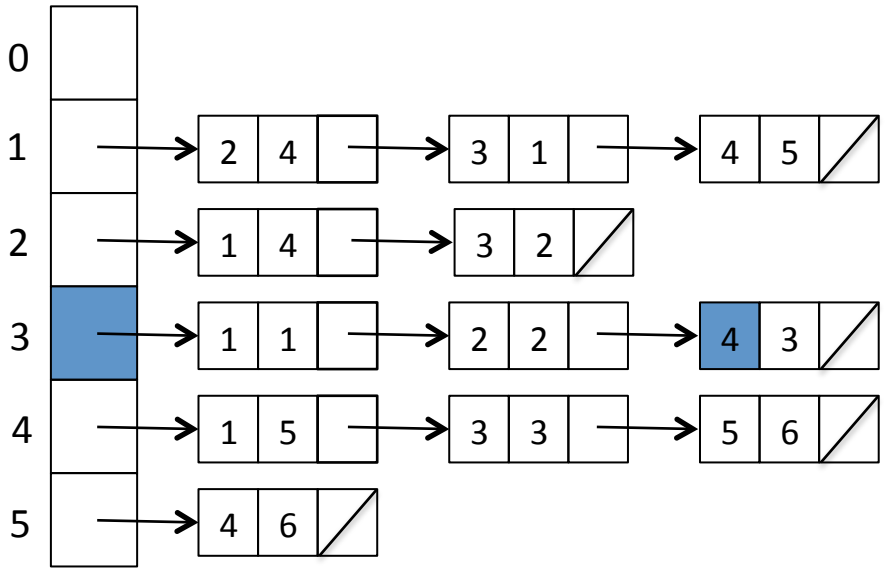



| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | F | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | F | 5 | 1 |
| 5 | F | ∞ | -1 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | ∞ |
| 1 | 3 | 1 | 4 |
| 2 | 4 | 5 | 1 |
| 3 | 1 | 1 | |
| | 2 | 2 | |

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

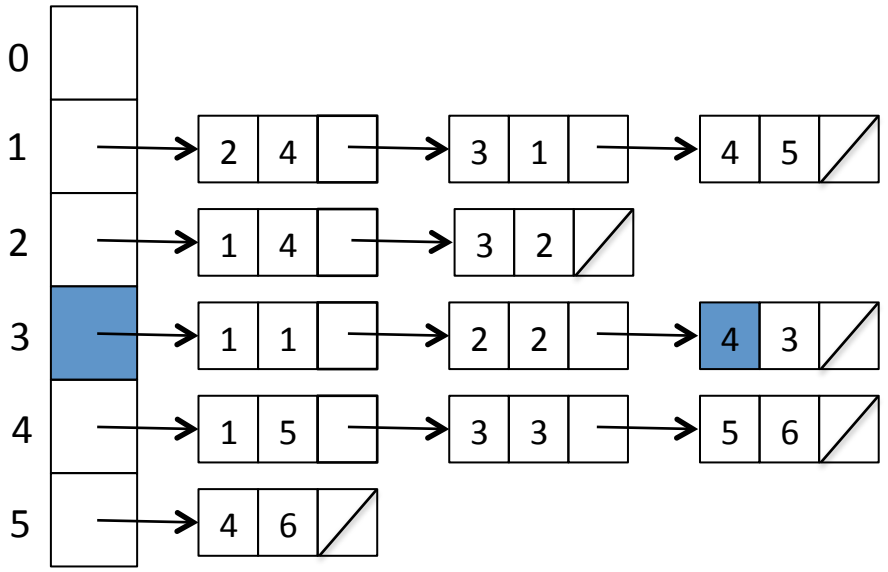
distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```




| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | F | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | F | 5 | 1 |
| 5 | F | ∞ | -1 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | ∞ |
| 1 | 3 | 1 | 4 |
| 2 | 4 | 5 | 1 |
| 3 | 1 | 1 | |
| | 2 | 2 | |
| | 4 | 3 | |

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

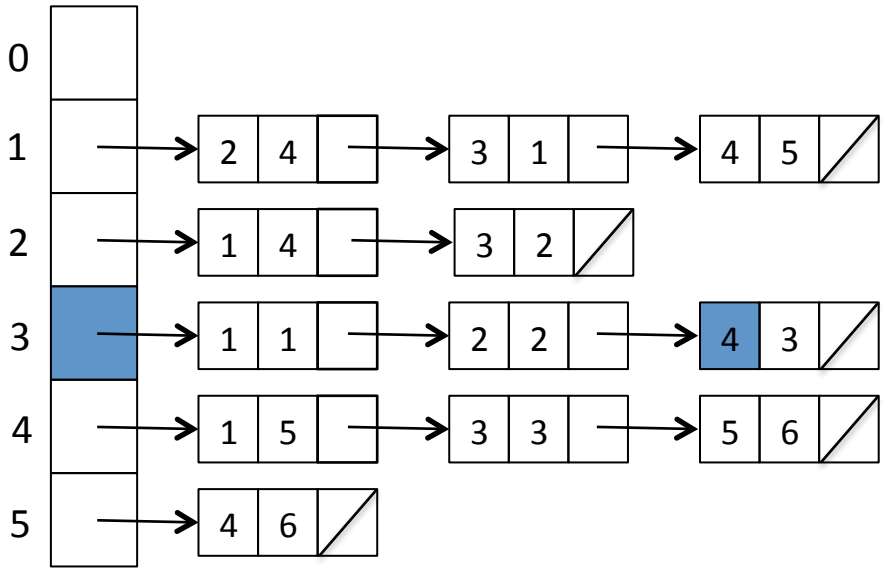
distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```




| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | F | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | F | 3 | 3 |
| 5 | F | ∞ | -1 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | ∞ |
| 1 | 3 | 1 | 4 |
| 2 | 4 | 5 | 1 |
| 3 | 1 | 1 | |
| | 2 | 2 | |
| | 4 | 3 | |

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

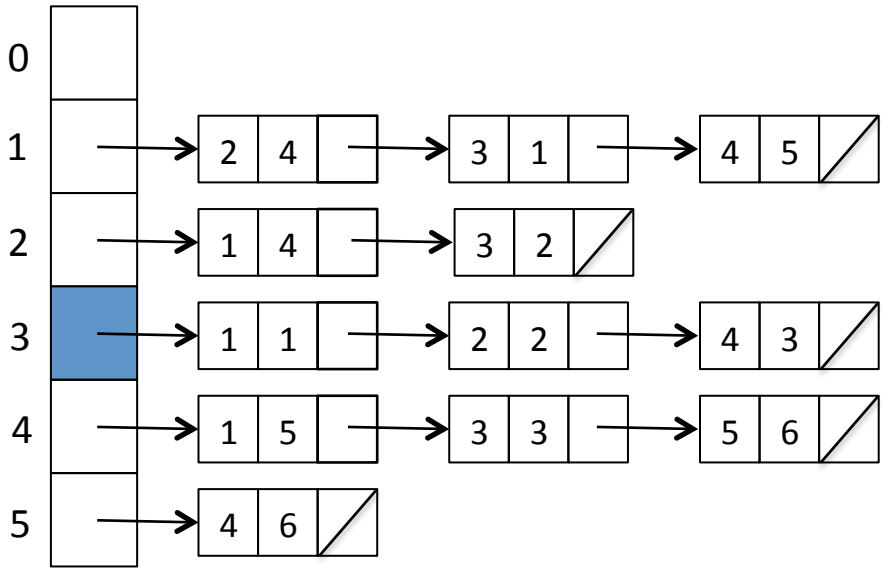
distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```




| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | F | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | F | 3 | 3 |
| 5 | F | ∞ | -1 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | ∞ |
| 1 | 3 | 1 | 4 |
| 2 | 4 | 5 | 1 |
| 3 | 1 | 1 | ∞ |
| 1 | 2 | 2 | 2 |
| 2 | 4 | 3 | |

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

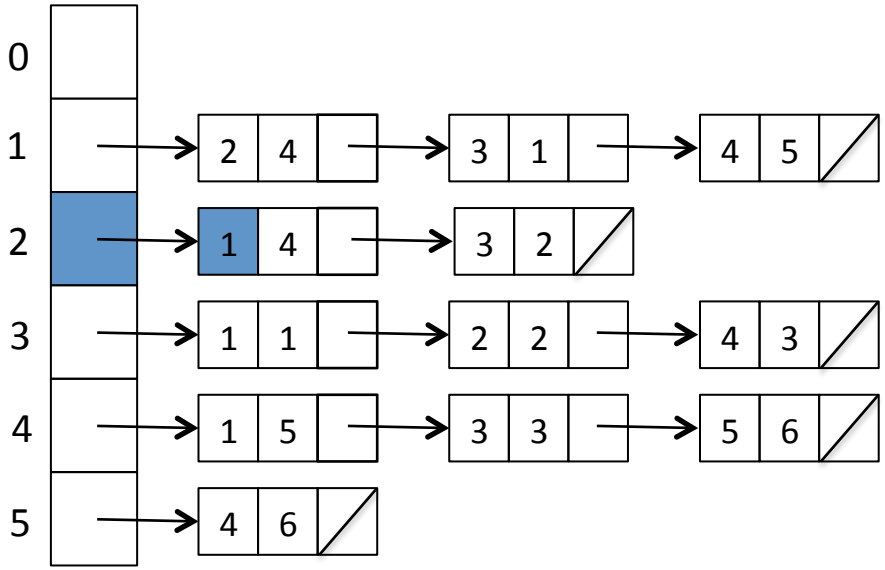
distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```




| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | T | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | F | 3 | 3 |
| 5 | F | ∞ | -1 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | ∞ |
| 1 | 3 | 1 | 4 |
| 2 | 4 | 5 | 1 |
| 3 | 1 | 1 | ∞ |
| 1 | 2 | 2 | 2 |
| 2 | 4 | 3 | |
| | 1 | 4 | |

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

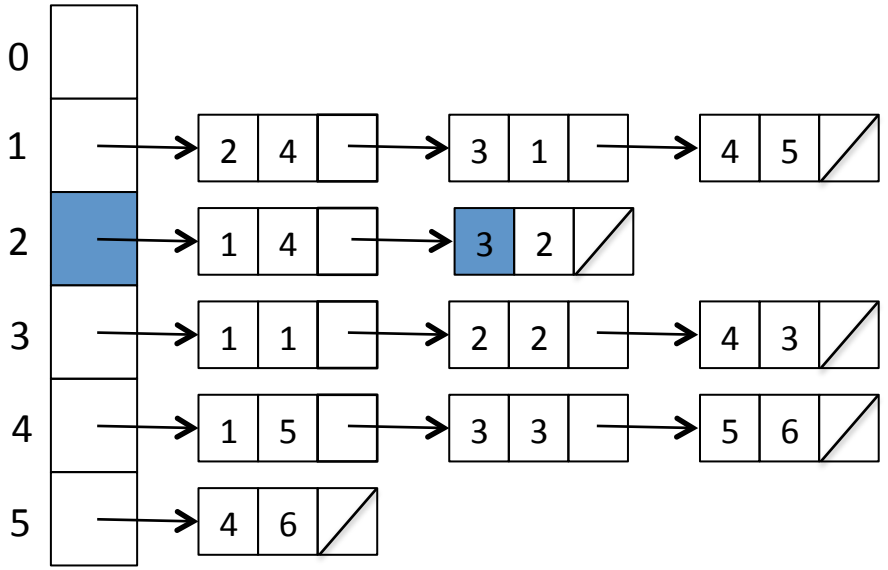
distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```




| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | T | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | F | 3 | 3 |
| 5 | F | ∞ | -1 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | ∞ |
| 1 | 3 | 1 | 4 |
| 2 | 4 | 5 | 1 |
| 3 | 1 | 1 | ∞ |
| 1 | 2 | 2 | 2 |
| 2 | 4 | 3 | |
| | 1 | 4 | |
| | 3 | 2 | |

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

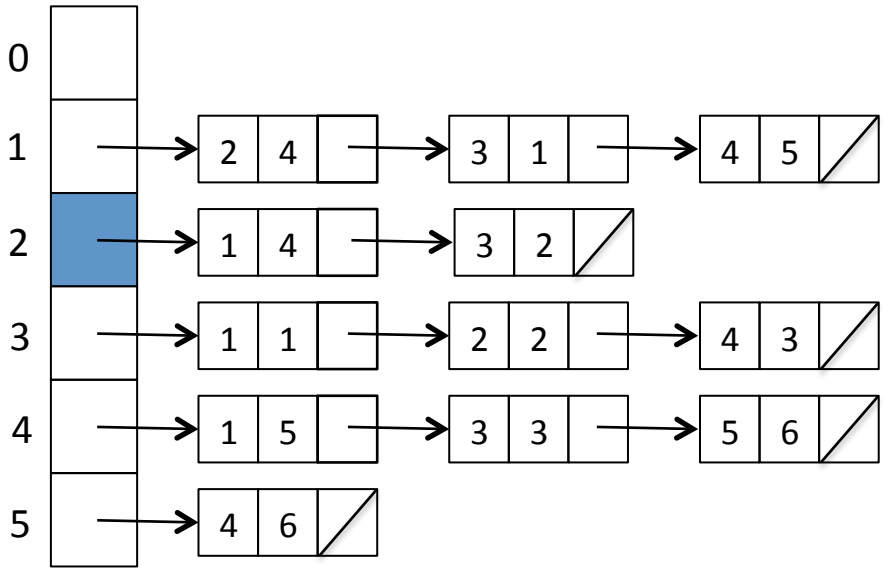
distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```




| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | T | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | F | 3 | 3 |
| 5 | F | ∞ | -1 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | ∞ |
| 1 | 3 | 1 | 4 |
| 2 | 4 | 5 | 1 |
| 3 | 1 | 1 | ∞ |
| 1 | 2 | 2 | 2 |
| 2 | 4 | 3 | ∞ |
| 1 | 1 | 4 | 3 |
| 4 | 3 | 2 | |

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

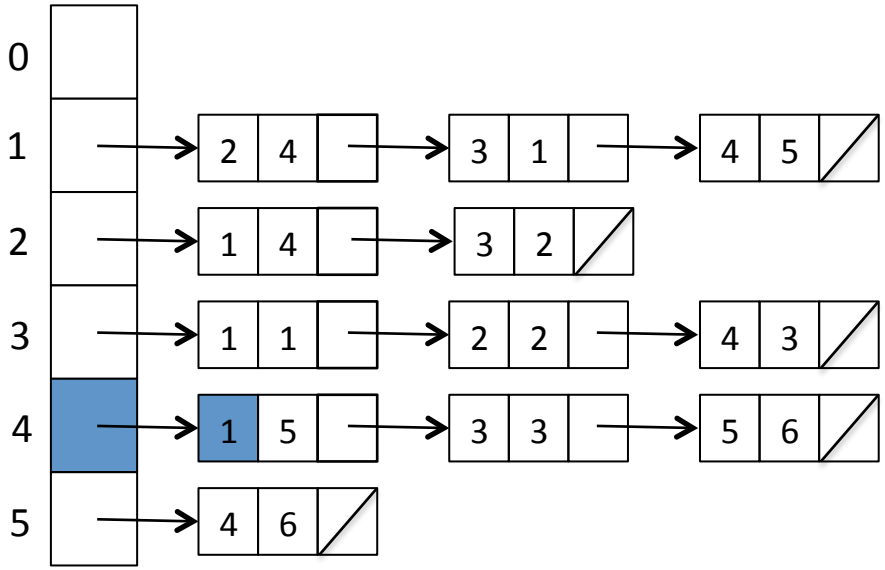
distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```




| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | T | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | T | 3 | 3 |
| 5 | F | ∞ | -1 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | ∞ |
| 1 | 3 | 1 | 4 |
| 2 | 4 | 5 | 1 |
| 3 | 1 | 1 | ∞ |
| 1 | 2 | 2 | 2 |
| 2 | 4 | 3 | ∞ |
| 1 | 1 | 4 | 3 |
| 4 | 3 | 2 | |
| | 1 | 5 | |

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

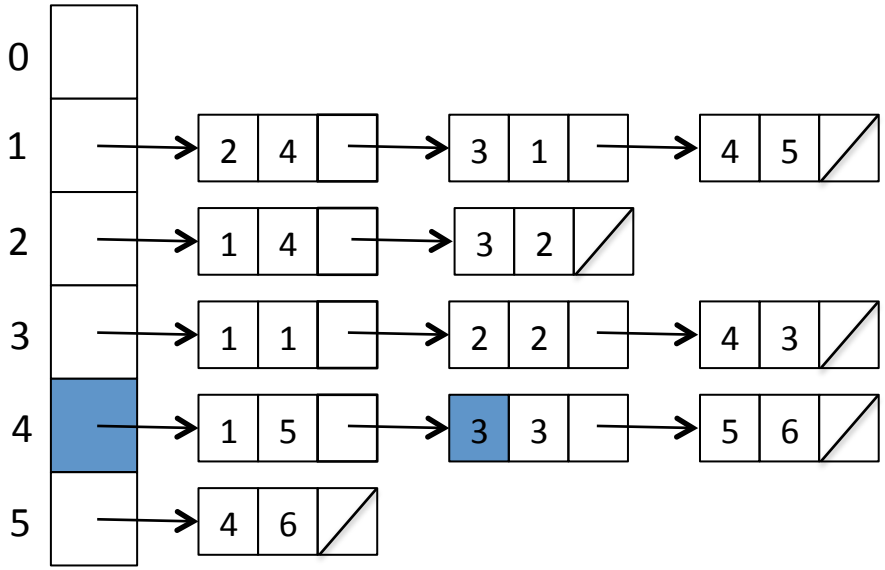
distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```




| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | T | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | T | 3 | 3 |
| 5 | F | ∞ | -1 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | ∞ |
| 1 | 3 | 1 | 4 |
| 2 | 4 | 5 | 1 |
| 3 | 1 | 1 | ∞ |
| 1 | 2 | 2 | 2 |
| 2 | 4 | 3 | ∞ |
| 1 | 1 | 4 | 3 |
| 4 | 3 | 2 | |
| | 1 | 5 | |
| | 3 | 3 | |

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

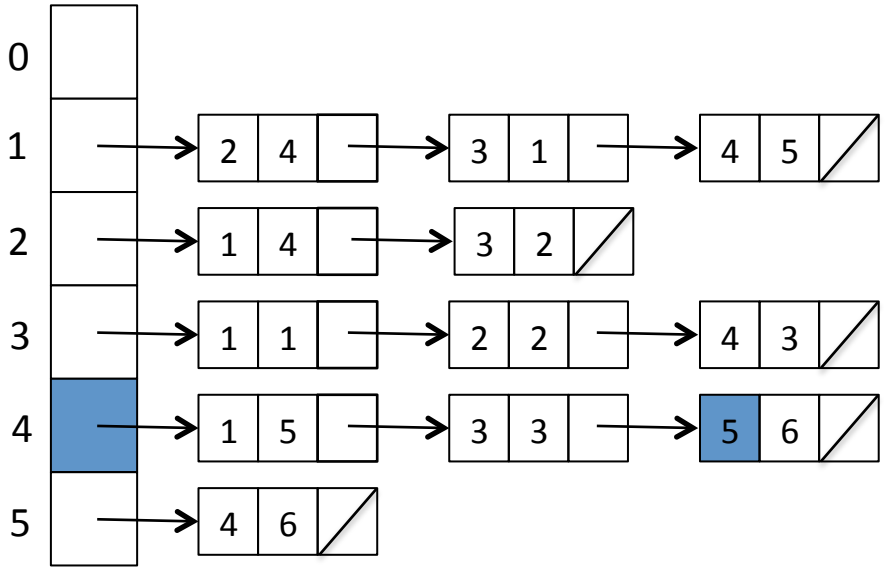
distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```




| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | T | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | T | 3 | 3 |
| 5 | F | ∞ | -1 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | ∞ |
| 1 | 3 | 1 | 4 |
| 2 | 4 | 5 | 1 |
| 3 | 1 | 1 | ∞ |
| 1 | 2 | 2 | 2 |
| 2 | 4 | 3 | ∞ |
| 1 | 1 | 4 | 3 |
| 4 | 3 | 2 | |
| | 1 | 5 | |
| | 3 | 3 | |
| | 5 | 6 | |

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

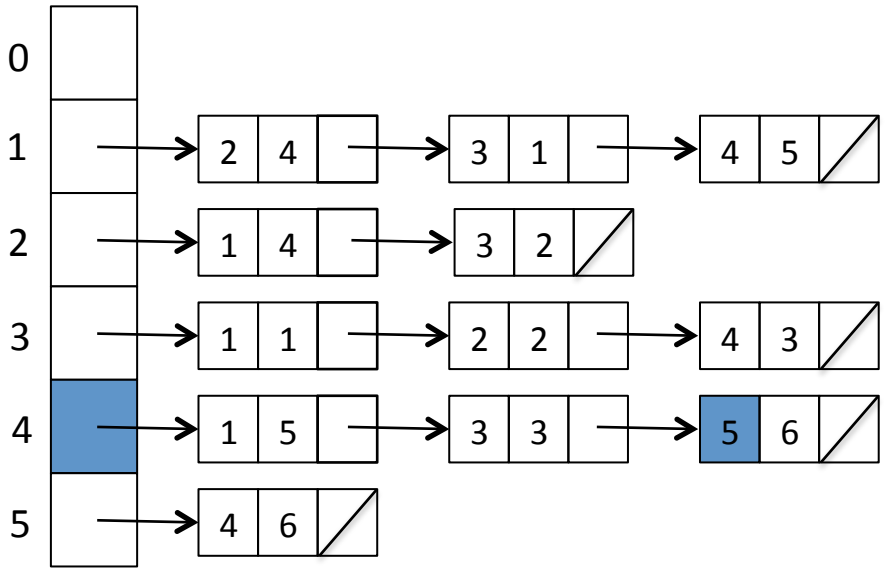
distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```




| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | T | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | T | 3 | 3 |
| 5 | F | 6 | 4 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | ∞ |
| 1 | 3 | 1 | 4 |
| 2 | 4 | 5 | 1 |
| 3 | 1 | 1 | ∞ |
| 1 | 2 | 2 | 2 |
| 2 | 4 | 3 | ∞ |
| 1 | 1 | 4 | 3 |
| 4 | 3 | 2 | |
| | 1 | 5 | |
| | 3 | 3 | |
| | 5 | 6 | |

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

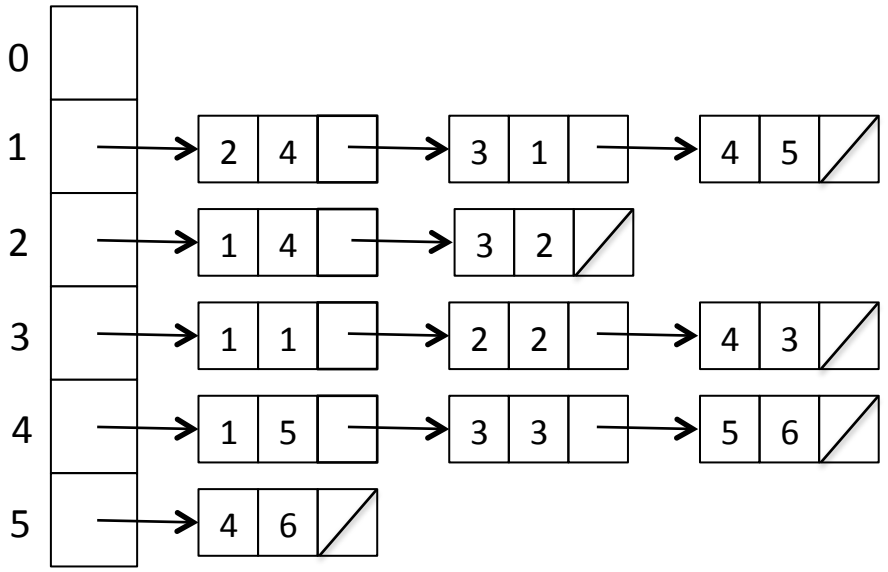
distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```




| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | T | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | T | 3 | 3 |
| 5 | F | 6 | 4 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | 2 | 4 | ∞ |
| 1 | 3 | 1 | 4 |
| 2 | 4 | 5 | 1 |
| 3 | 1 | 1 | ∞ |
| 1 | 2 | 2 | 2 |
| 2 | 4 | 3 | ∞ |
| 1 | 1 | 4 | 3 |
| 4 | 3 | 2 | ∞ |
| 1 | 1 | 5 | 6 |
| 5 | 3 | 3 | |
| | | 5 | 6 |

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

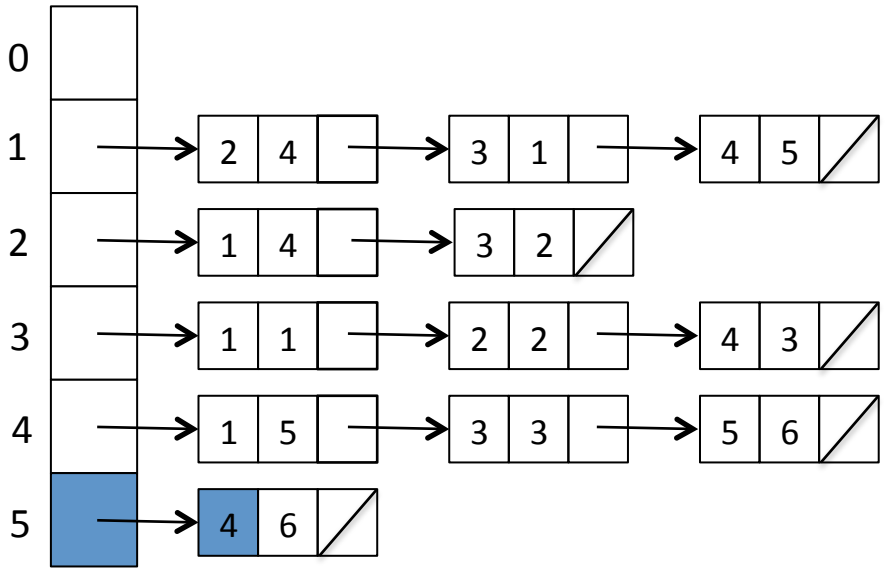
distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```




| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | T | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | T | 3 | 3 |
| 5 | T | 6 | 4 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | ... | ... | ∞ |
| 1 | 4 | 6 | 4 |
| 2 | | | 1 |
| 3 | | | ∞ |
| 1 | | | 2 |
| 2 | | | ∞ |
| 1 | | | 3 |
| 4 | | | ∞ |
| 1 | | | 6 |
| 5 | | | |

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

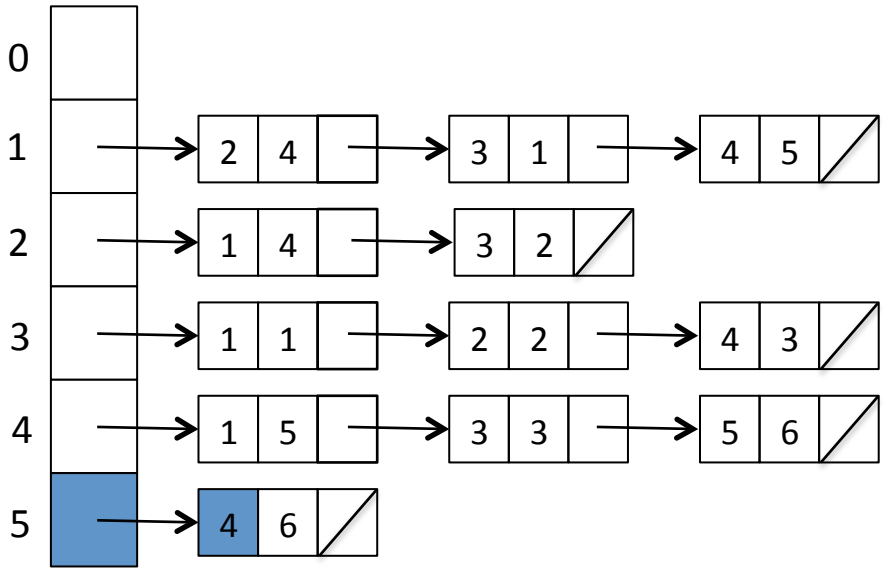
distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```

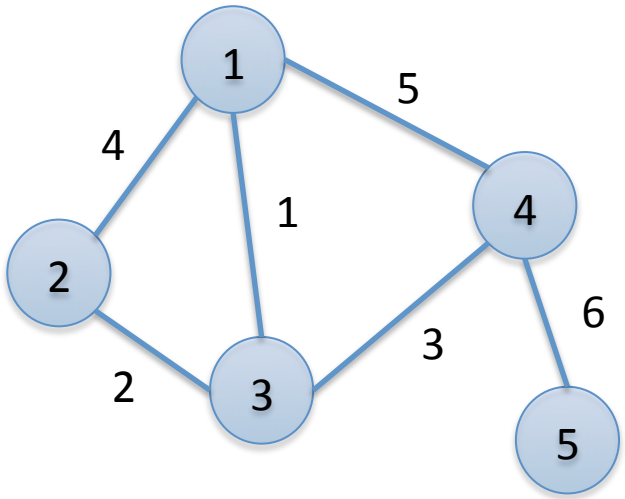


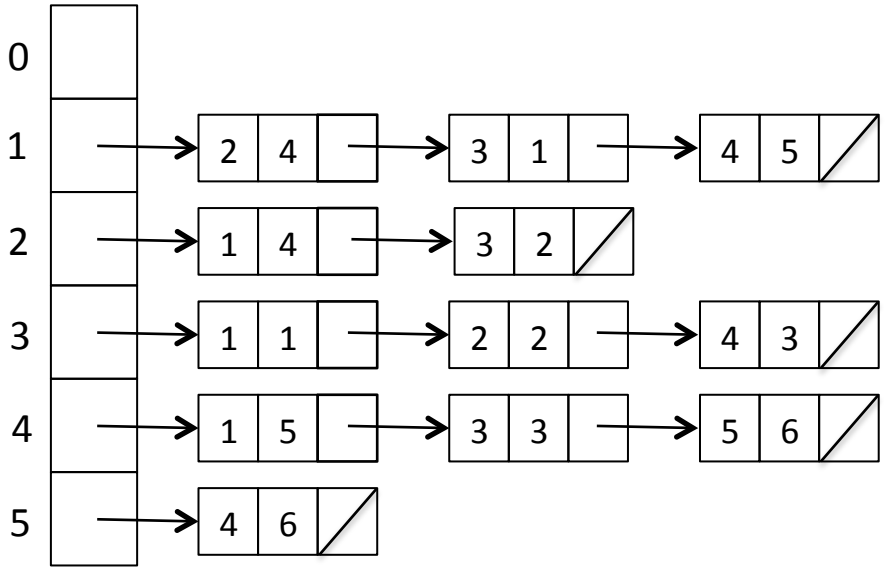

| | intree | distance | parent |
|---|---|---|---|
| 0 | | | |
| 1 | T | 0 | -1 |
| 2 | T | 2 | 3 |
| 3 | T | 1 | 1 |
| 4 | T | 3 | 3 |
| 5 | T | 6 | 4 |

| v | w | weight | dist |
|---|---|---|---|
| 1 | ... | ... | ∞ |
| 1 | 4 | 6 | 4 |
| 2 | | | 1 |
| 3 | | | ∞ |
| 1 | | | 2 |
| 2 | | | ∞ |
| 1 | | | 3 |
| 4 | | | ∞ |
| 1 | | | 6 |
| 5 | | | ∞ |
| 1 | | | |

# Minimum Spanning Tree

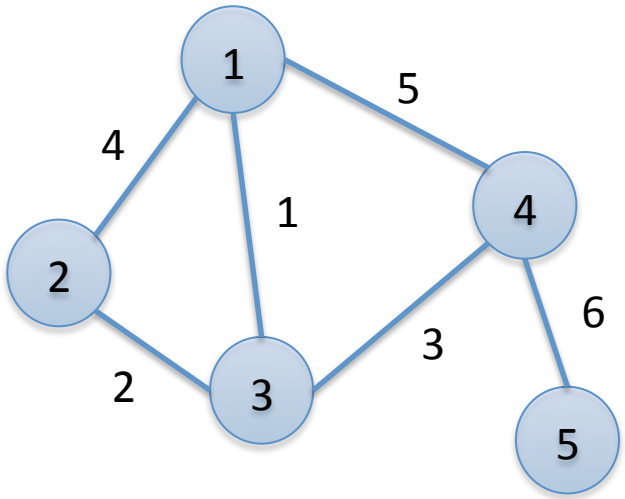
- Prim's Algorithm
  - The minimum spanning tree itself or its cost has to be reconstructed
    - Augment this procedure with statements that print the edges as they are found or totals the weight of all selected edges
    - Alternatively, the tree topology is given by the the `parent` relations and so it can be constructed using the original graph

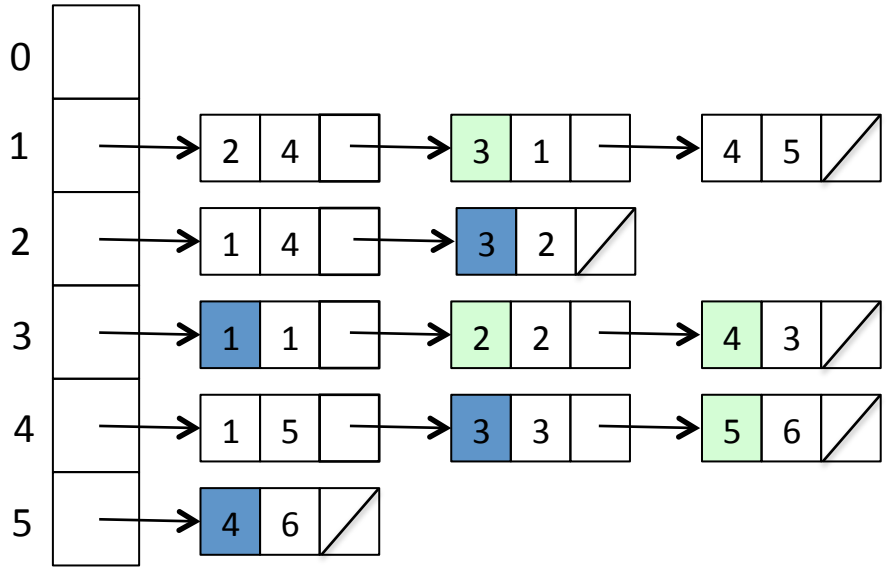


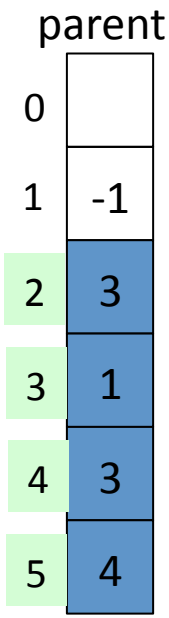



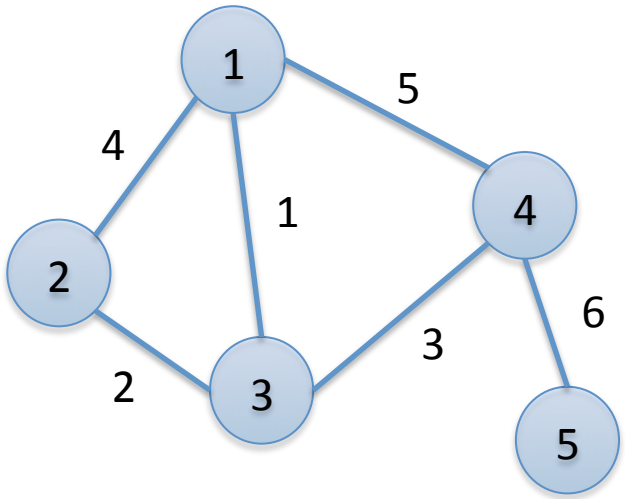

parent

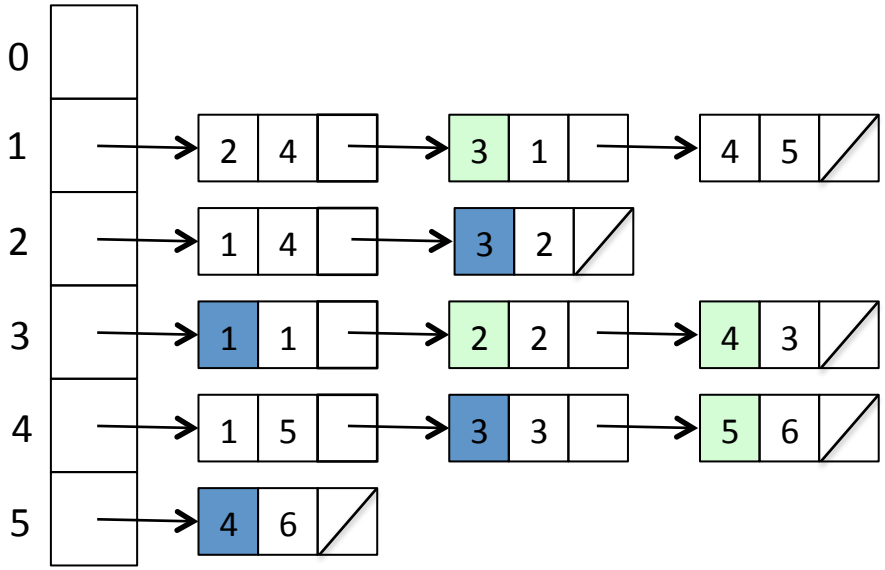
| | |
|---|---|
| 0 | |
| 1 | -1 |
| 2 | 3 |
| 3 | 1 |
| 4 | 3 |
| 5 | 4 |

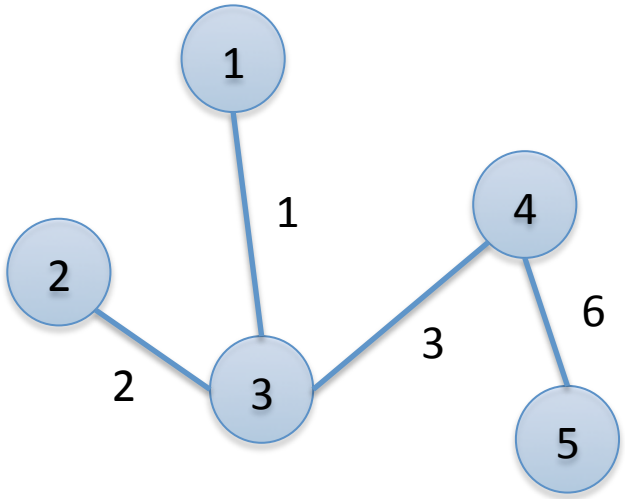


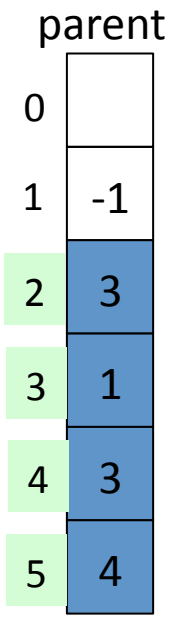





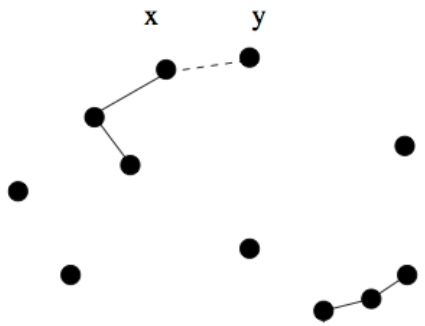



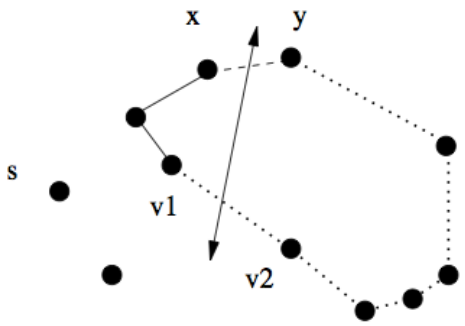



# Minimum Spanning Tree

- Kruskal's Algorithm
  - Greedy algorithm:
    - Builds up connected components of vertices, culminating in the MST
    - Initially each vertex forms its own connected component
    - The algorithm repeatedly considers the lightest remaining edge
      - If its two endpoints lie within the same connected component, discard it (because adding it would create a cycle in the tree)
      - Otherwise, insert the edge and merge the two components in to one



# Minimum Spanning Tree

- Kruskal's Algorithm

```
Kruskal-MST(G):  // note: no start vertex s; cf Prim's Algorithm

Put the edges in a priority queue ordered by weight
count = 0
While (count < n-1) do
    get next edge (v, w)
    if (component (v) != component (w))
        add to T_Kruskal
        merge component (v) and component (w)
```

# Minimum Spanning Tree

- Prim's Algorithm vs. Kruskal's Algorithm
  - Prim $O(mn)$ … if we don't keep track of cheapest edge
  - Prim $O(n^2)$ … if we do but use a simple data structure
  - Prim $O(m + n \log n)$ … if we use a priority queue data structure
  - Kruskal $O(m \log m)$

    $n$ vertices, $m$ edges

  - Prim's algorithm is faster on dense graphs (depends on $n$)
  - Kruskal's algorithm is faster on sparse graphs (depends on $m$)