Algorithms and Data Structures
CS-CO-412

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Graphs

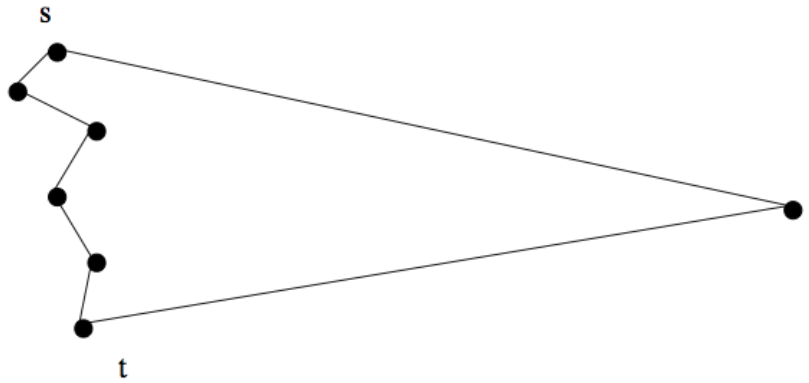
Lecture 14

Topic Overview

- Graphs
  - Types of graph
  - Adjacency matrix representation
  - Adjacency list representation
  - Breadth-First Search traversal
  - Depth-First Search traversal
  - Topological Sorting
  - Minimum Spanning Tree
    - Prim's Algorithm
    - Kruskal's algorithm
  - **Shortest Path Algorithms**
    - **Dijkstra's algorithm**
    - **Floyd's algorithm**

Shortest Paths

- A *path* is a sequence of edges connecting two vertices
- The shortest path in an unweighted graph can be constructed using Breadth-First Search
- Also works if the weights are all the same in a weighted graph
- Does not work for weighted graphs in general:
  the shortest route (e.g. in time) may pass through many intermediate vertices



Shortest Paths

- Two main algorithms for finding shortest paths
  - Dijkstra's Algorithm
    - Finds shortest path between start and destination vertices
      - Some implementations find the shortest path between a start vertex and all other vertices, i.e. shortest path spanning tree rooted in the start vertex
    - $O(n^2)$ for with simple data structures
  - Floyd's Algorithm
    - Finds shortest path between *all pairs* of vertices in a graphs
    - $O(n^3)$

Shortest Paths

- Dijkstra's Algorithm
  - Greedy algorithm
  - Repeatedly select the smallest weight edge that will extend the path
    - Start from given vertex,
    - Extend the path, one edge at a time,
    - Until all vertices are included
  - Very similar to Prim's algorithm, except …
    - Instead of just considering the weight of the next potential edge
    - **We also have to consider the distance from the start to the vertex from which that edge emanates**

Shortest Paths

ShortestPath-Dijkstra(G, s, t)

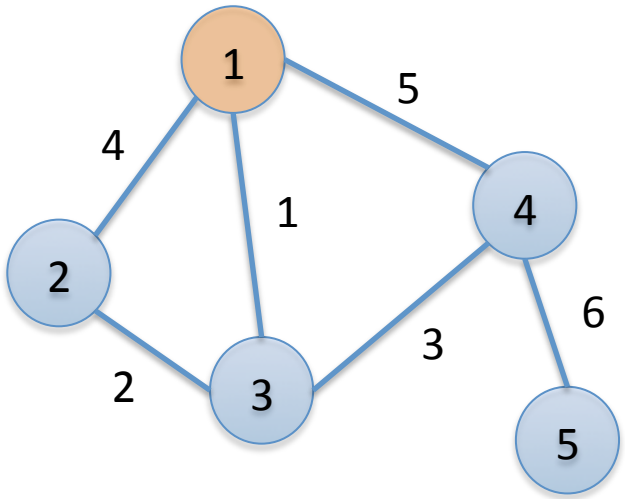
path= {s}
for i = 1 to n, dist[i] = ∞
for each edge (s, v), dist[v] = w(s, v) // initial distances are just the weights
last = s
while (last != t)
    select v_{next}, the unknown vertex minimizing dist[v]
    for each edge (v_{next}, x), dist[x] = min[dist[x], dist[vnext] + w(v_{next}, x)]
    last = v_{next}
    path = path ∪ {v_{next}}

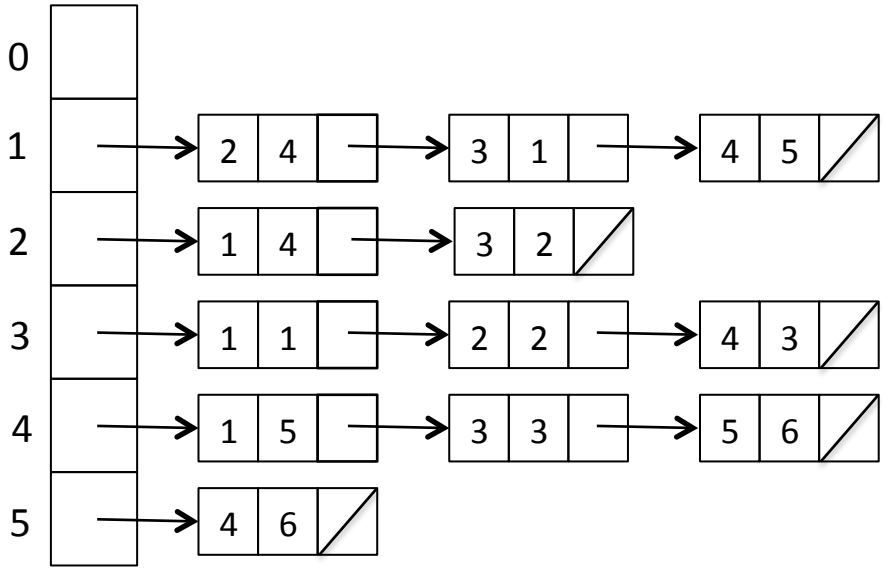
Shortest Paths

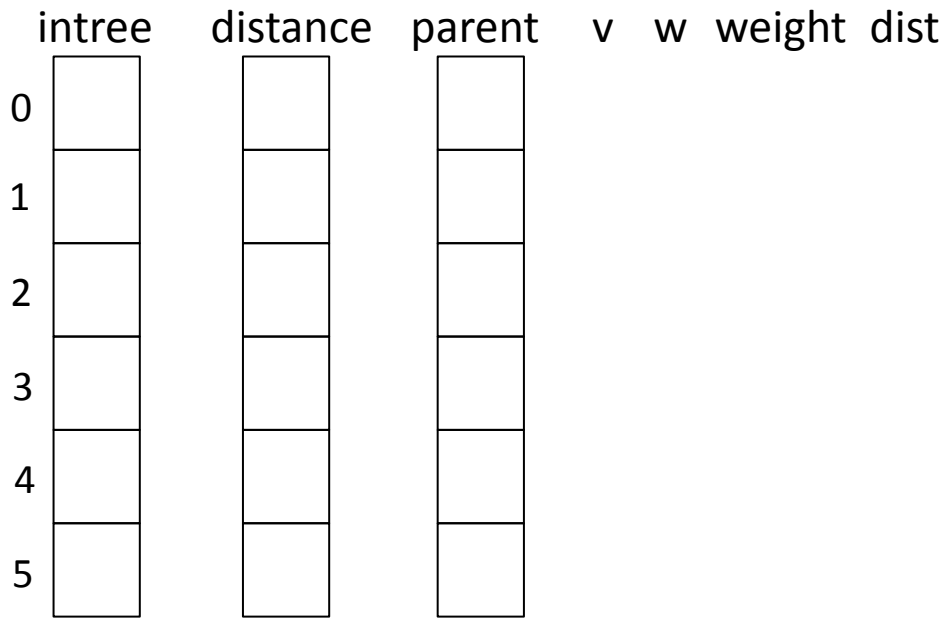
```
/* Dijkstra's algorithm: implementation based of Prim's algorithm */

dijkstra(graph *g, int start) {
   int i;                  /* counter                          */
   edgenode *p;            /* temporary pointer                */
   bool intree[MAXV+1];    /* is the vertex in the tree yet?   */
   int distance[MAXV+1];   /* cost of adding to tree           */
   int parent[MAXV+1];     /* parent vertex                    */
   int v;                  /* current vertex to process        */
   int w;                  /* candidate next vertex            */
   int weight;             /* edge weight                      */
   int dist;               /* best current distance from start */

   for (i=1; i<=g->nvertices; i++) {
      intree[i] = FALSE;
      distance[i] = MAXINT;
      parent[i] = -1;
   }

   distance[start] = 0;
   v = start;
```

Shortest Paths

```
    while (intree[v] == FALSE) {
       intree[v] = TRUE;
       p = g->edges[v];
       while (p != NULL) {
          w = p->y;
          weight = p->weight;
          if ((distance[v]+weight < distance[w])) { //changes from Prim
             distance[w] = distance[v]+weight ;
             parent[w] = v;
          }
          p = p->next;
       }
       v = 1;
       dist = MAXINT;
       for (i=1; i<=g->nvertices; i++)
          if ((intree[i] == FALSE) && (distance[i] < dist)) {
             dist = distance[i];
             v = i;
          }
    }
}
```







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   intree[i] = FALSE;
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}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((distance[v]+weight < distance[w])) {
         distance[w] = distance[v]+weight;
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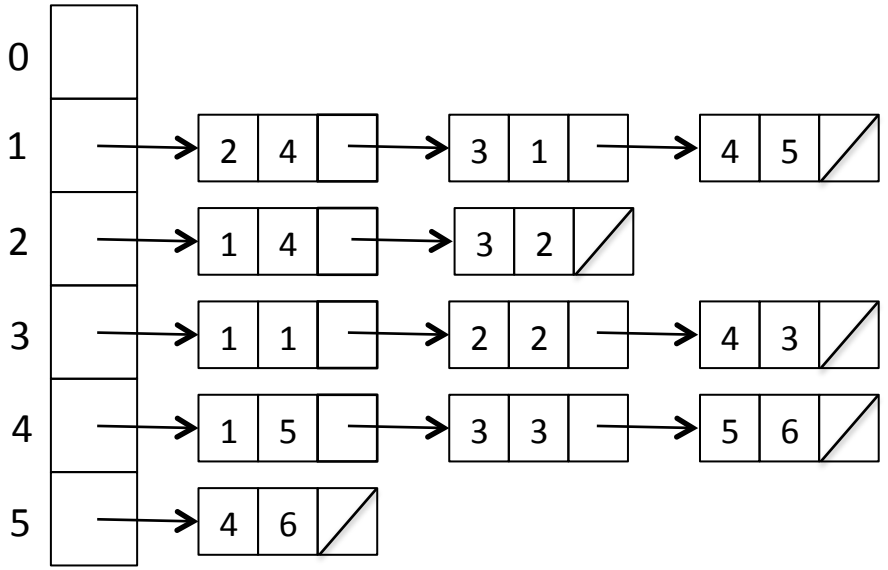
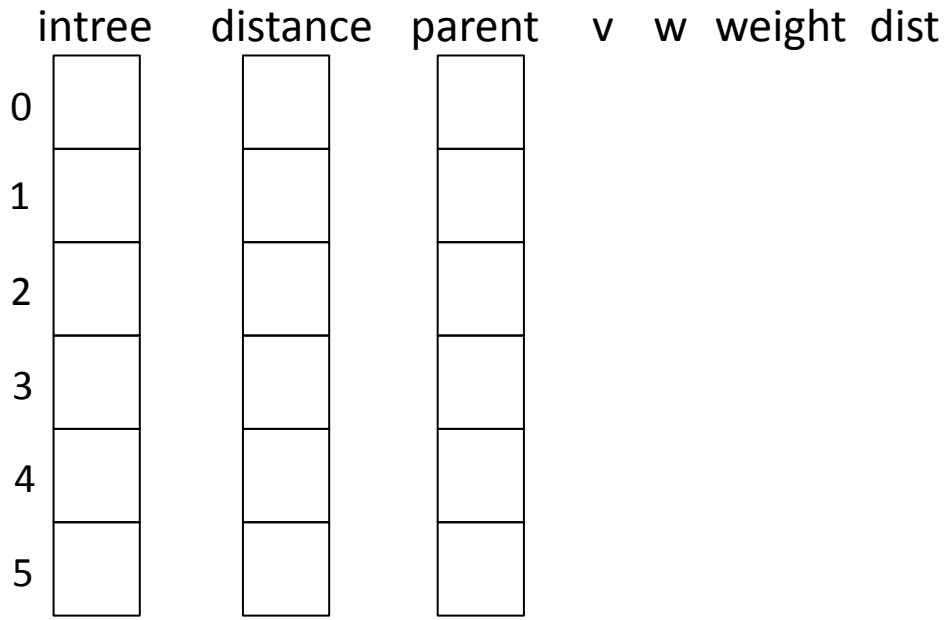
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```





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         dist = distance[i];
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}
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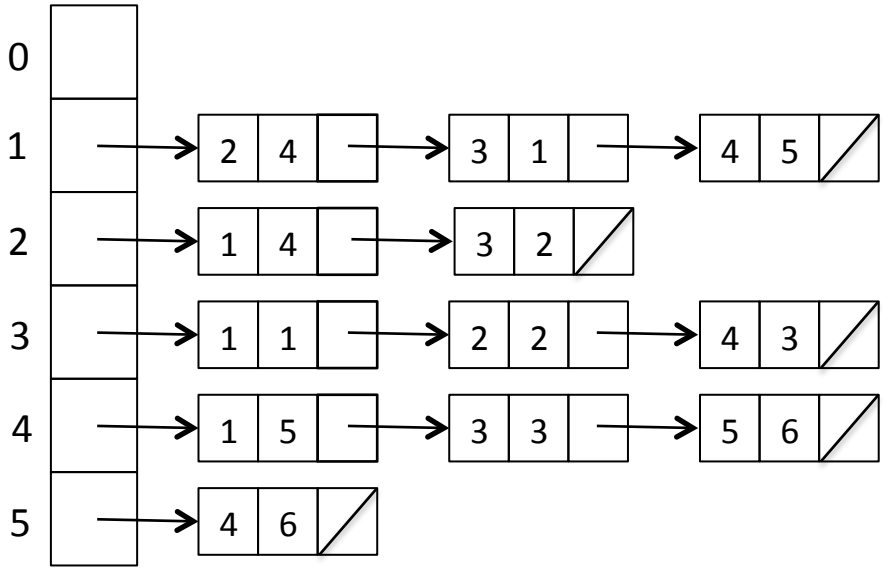

	intree	distance	parent
0			
1	F	∞	-1
2	F	∞	-1
3	F	∞	-1
4	F	∞	-1
5	F	∞	-1

v w weight dist

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
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}

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   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((distance[v]+weight < distance[w])) {
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      }
      p = p->next;
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   v = 1;
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   for (i=1; i<=g->nvertices; i++)
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         dist = distance[i];
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}
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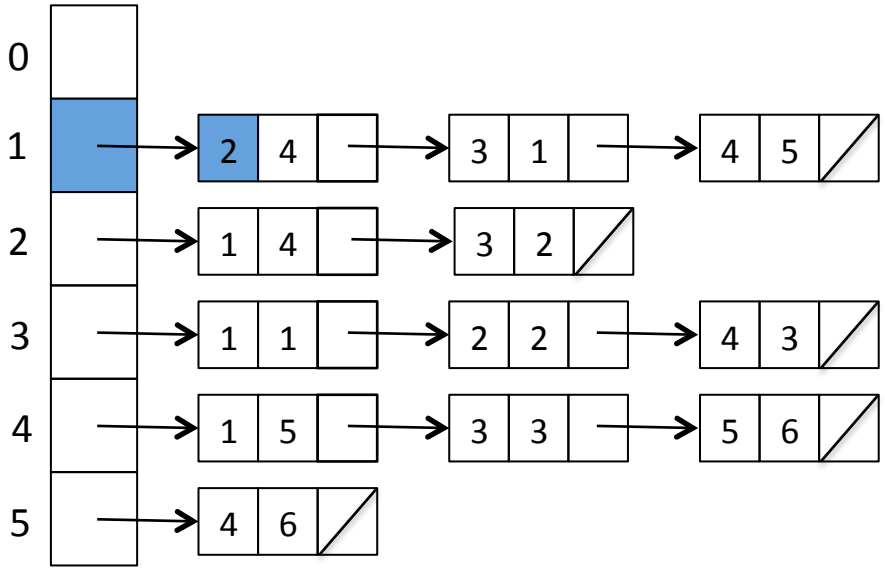

	intree	distance	parent
0			
1	T	0	-1
2	F	∞	-1
3	F	∞	-1
4	F	∞	-1
5	F	∞	-1

v	w	weight	dist
1	2	4	

```
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         distance[w] = distance[v]+weight ;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```

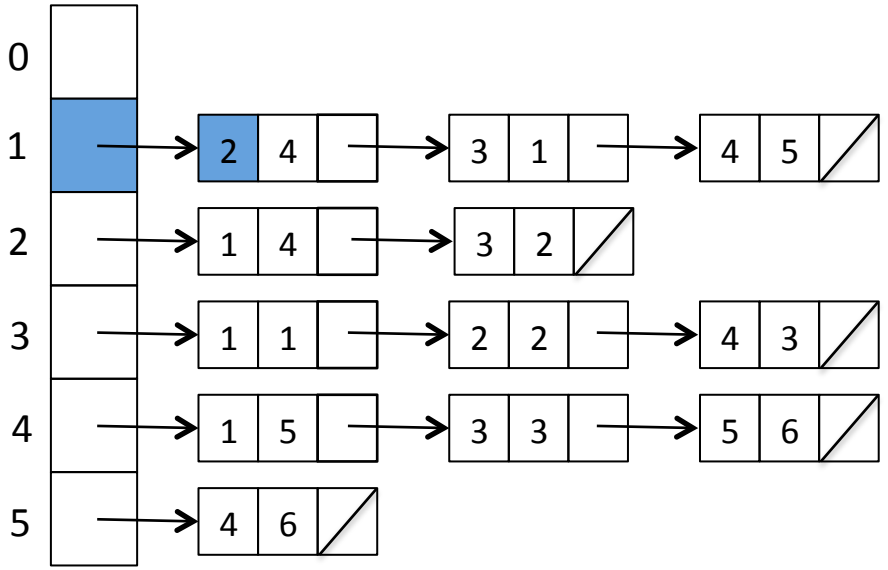

	intree	distance	parent
0			
1	T	0	-1
2	F	4	1
3	F	∞	-1
4	F	∞	-1
5	F	∞	-1

v	w	weight	dist
1	2	4	

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for (i=1; i<=g->nvertices; i++) {
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         v = i;
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}
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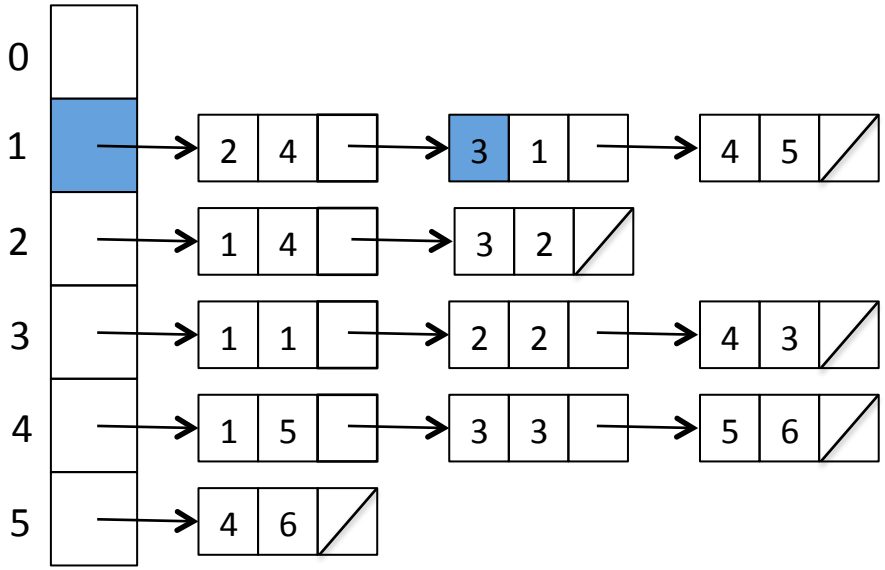

	intree	distance	parent
0			
1	T	0	-1
2	F	4	1
3	F	1	1
4	F	∞	-1
5	F	∞	-1

v	w	weight	dist
1	2	4	
	3	1	

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
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}

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while (intree[v] == FALSE) {

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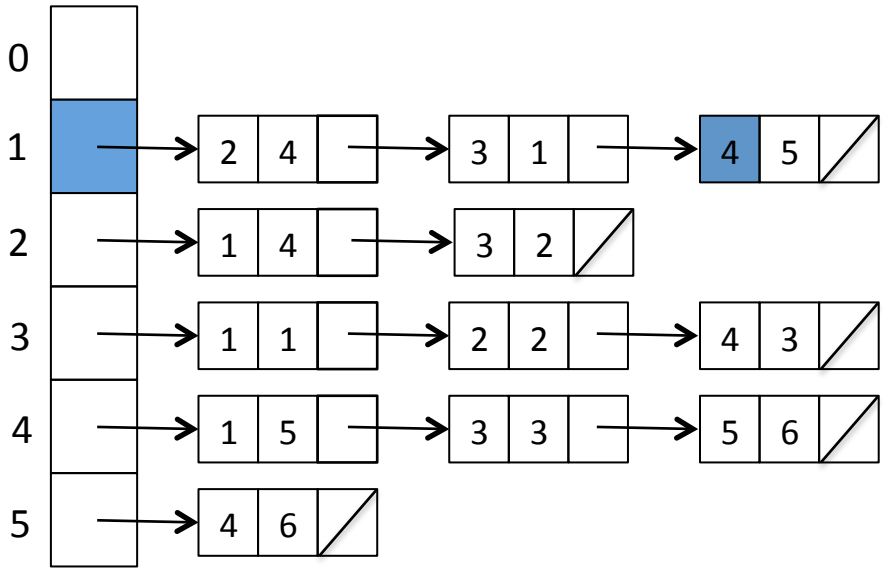

	intree	distance	parent
0			
1	T	0	-1
2	F	4	1
3	F	1	1
4	F	5	1
5	F	∞	-1

v	w	weight	dist
1	2	4	
	3	1	
	4	5	

```
for (i=1; i<=g->nvertices; i++) {
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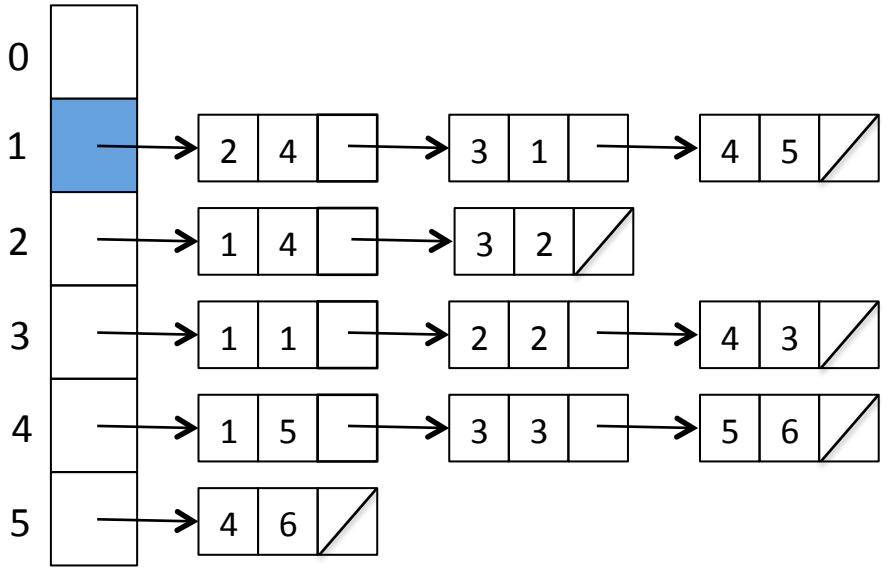
```



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v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3			

```
for (i=1; i<=g->nvertices; i++) {
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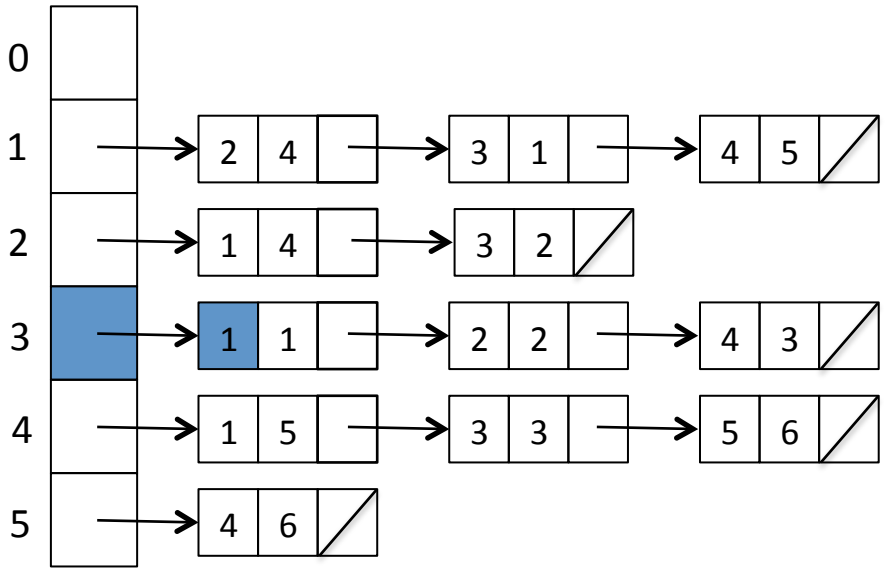
```



	intree	distance	parent
0			
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3	T	1	1
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v	w	weight	dist
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1	3	1	4
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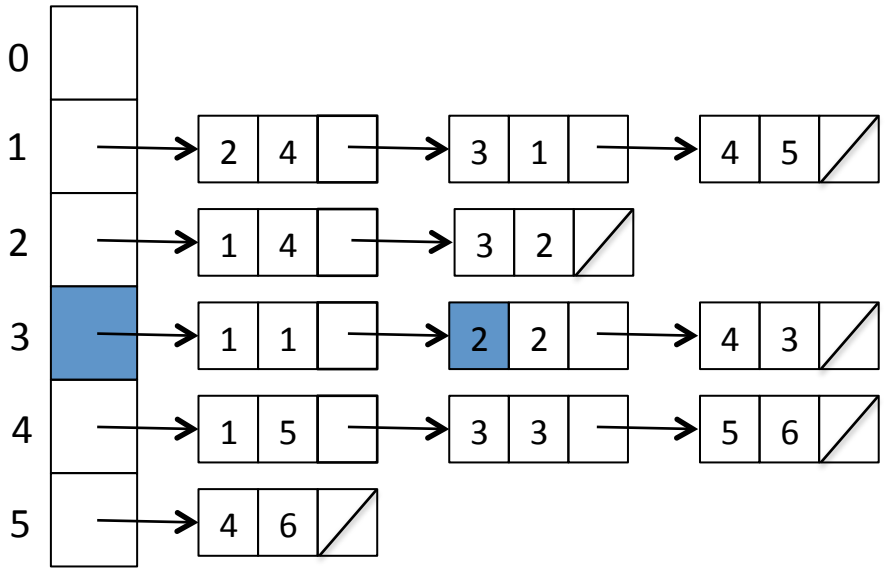

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	2	2	

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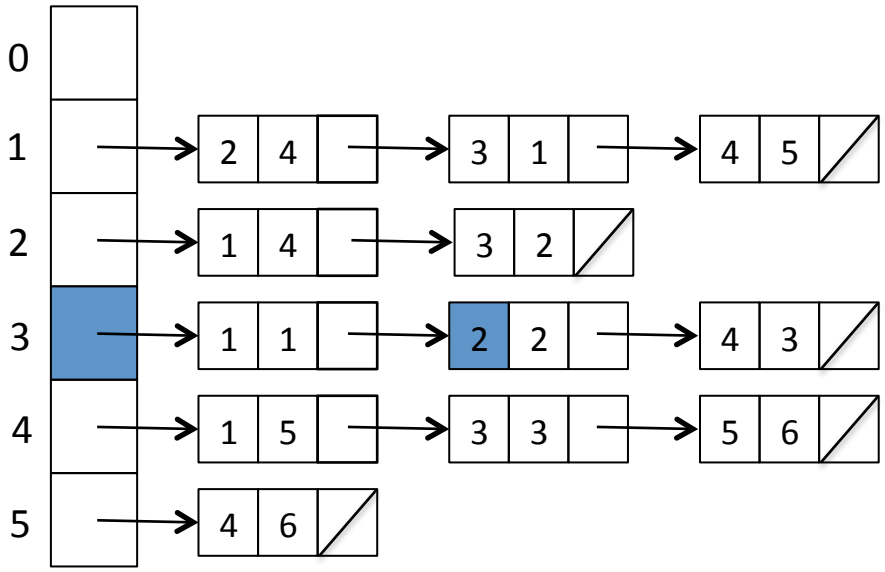

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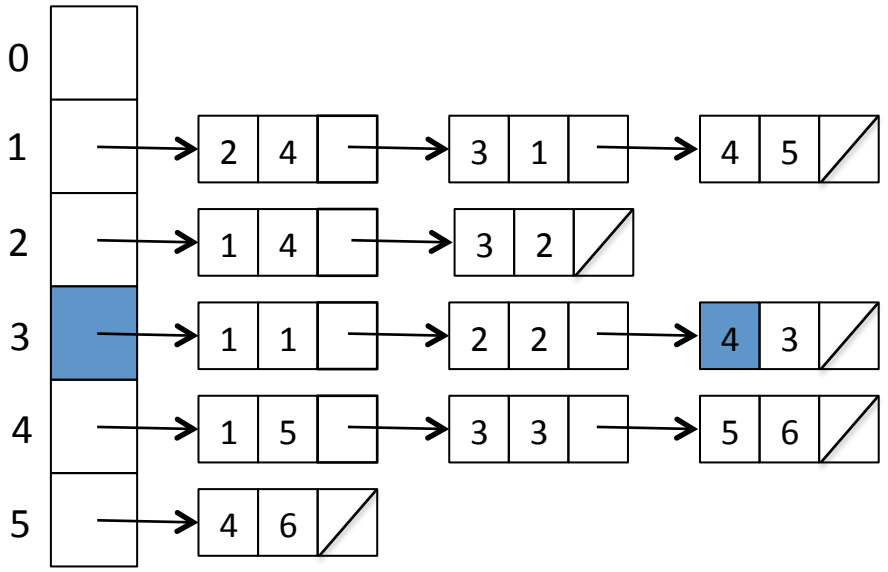

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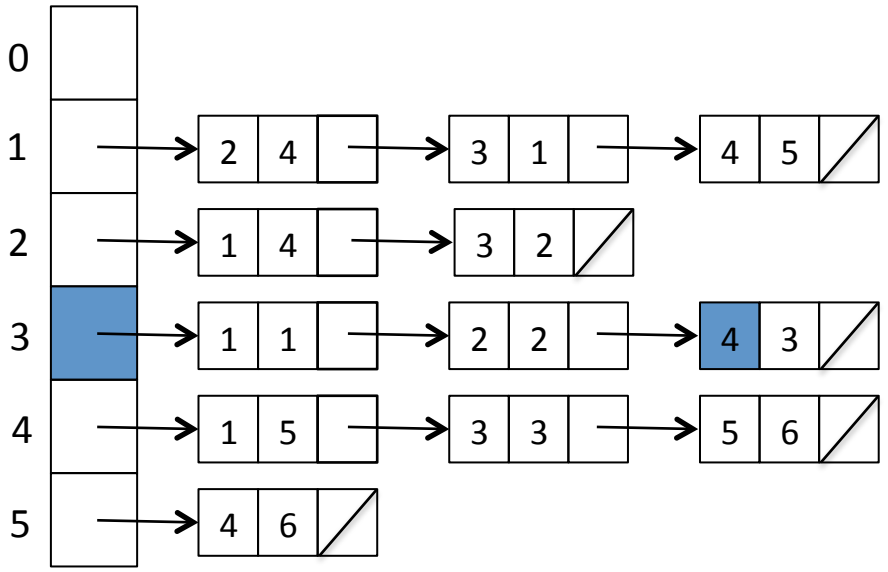

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v	w	weight	dist
1	2	4	∞
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2	4	5	1
3	1	1	
	2	2	
	4	3	

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}
```
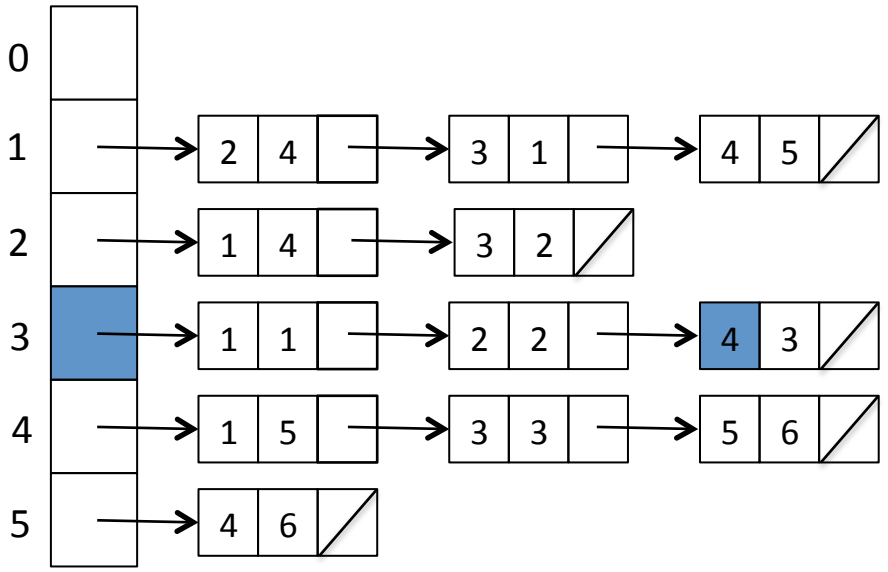
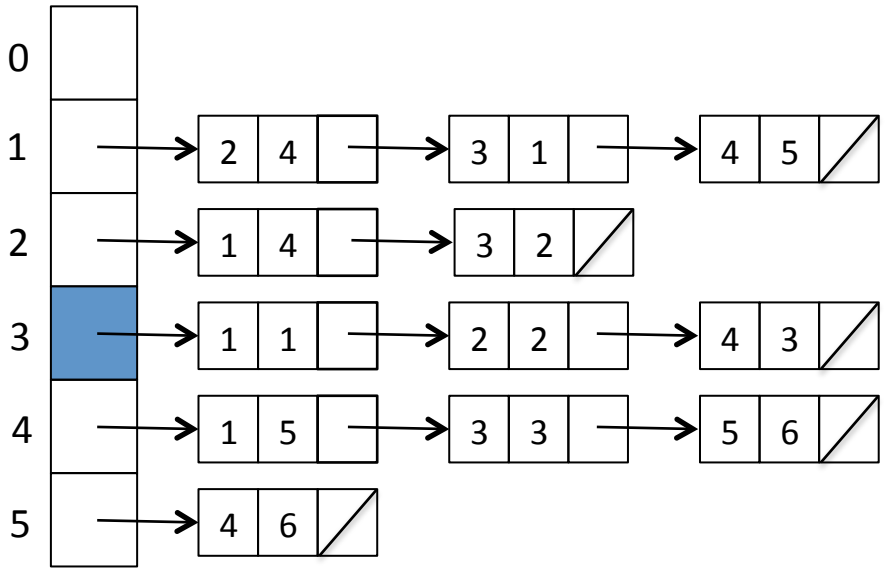



	intree	distance	parent
0			
1	T	0	-1
2	F	3	3
3	T	1	1
4	F	4	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	
	2	2	
	4	3	

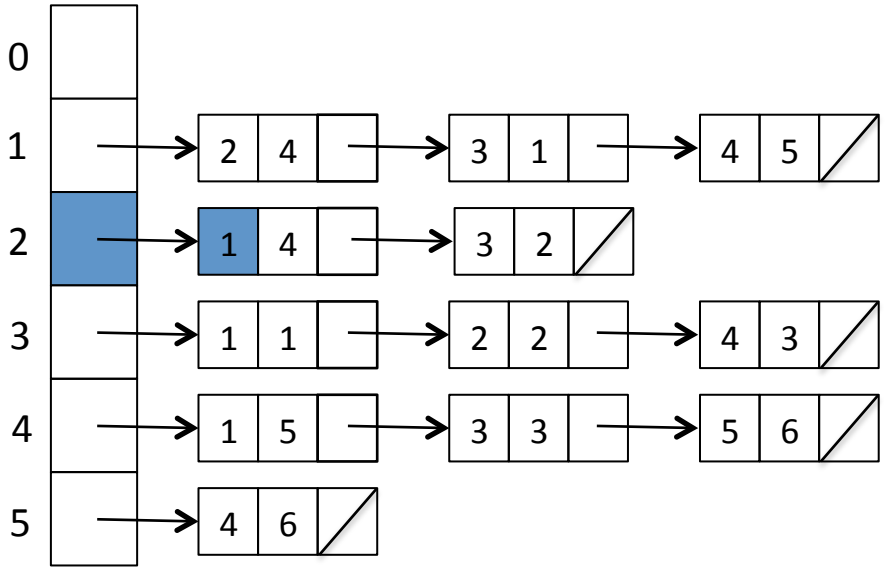
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((distance[v]+weight < distance[w])) {
         distance[w] = distance[v]+weight ;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	F	3	3
3	T	1	1
4	F	4	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	

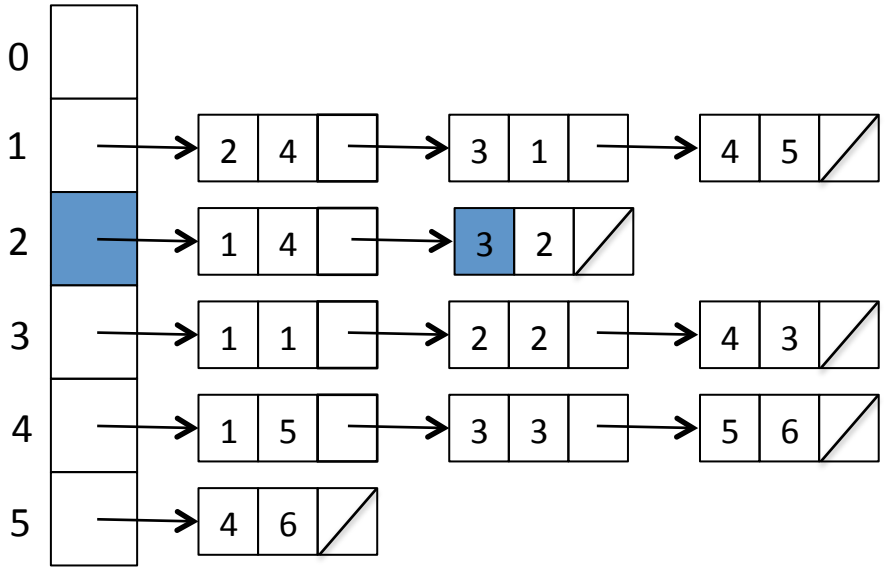
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((distance[v]+weight < distance[w])) {
         distance[w] = distance[v]+weight ;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	3	3
3	T	1	1
4	F	4	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	
	1	4	

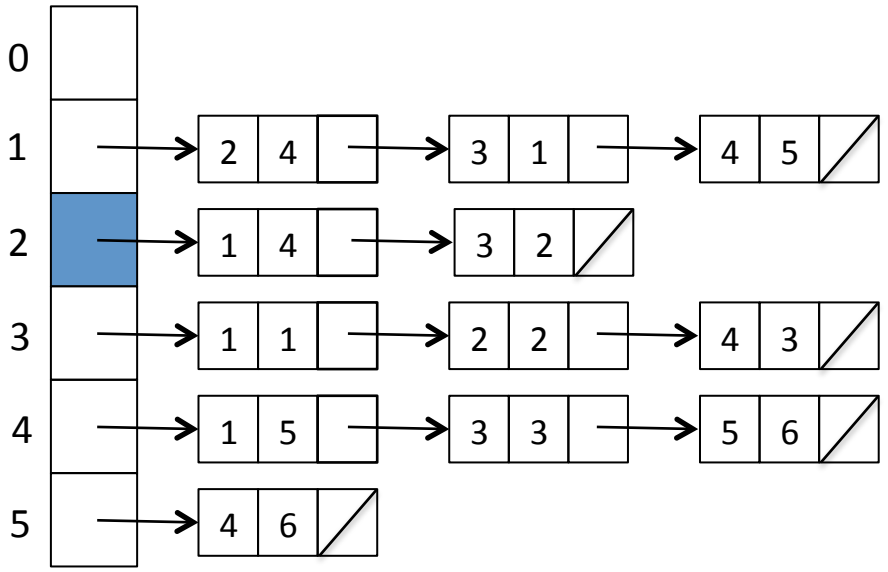
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((distance[v]+weight < distance[w])) {
         distance[w] = distance[v]+weight ;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	3	3
3	T	1	1
4	F	4	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	
	1	4	
	3	2	

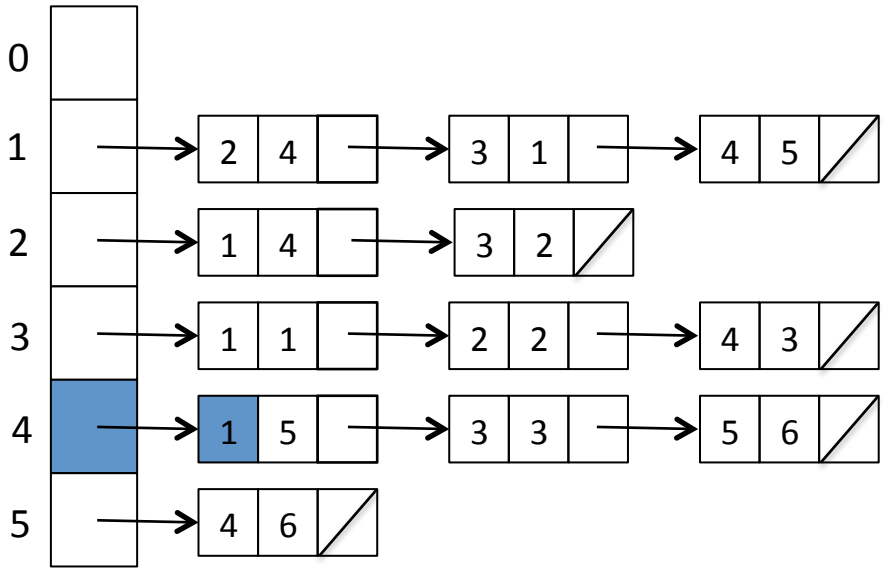
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((distance[v]+weight < distance[w])) {
         distance[w] = distance[v]+weight ;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	3	3
3	T	1	1
4	F	4	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	∞
1	1	4	3
4	3	2	

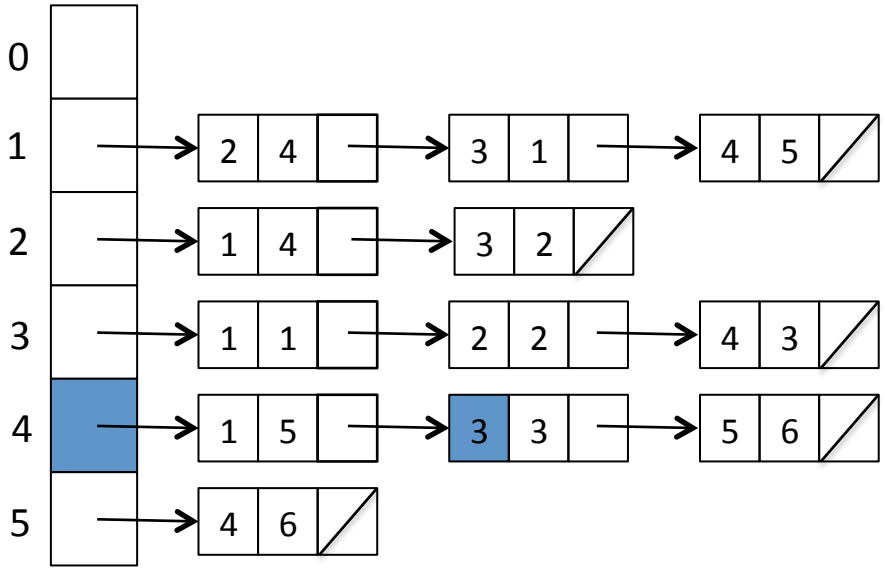
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((distance[v]+weight < distance[w])) {
         distance[w] = distance[v]+weight ;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	3	3
3	T	1	1
4	T	4	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	∞
1	1	4	3
4	3	2	
	1	5	

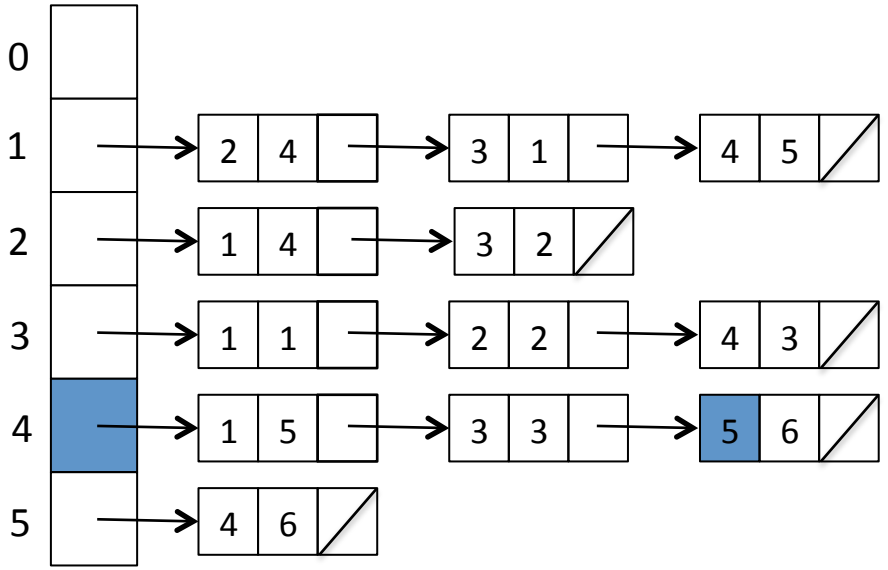
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((distance[v]+weight < distance[w])) {
         distance[w] = distance[v]+weight ;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	3	3
3	T	1	1
4	T	4	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	∞
1	1	4	3
4	3	2	
	1	5	
	3	3	

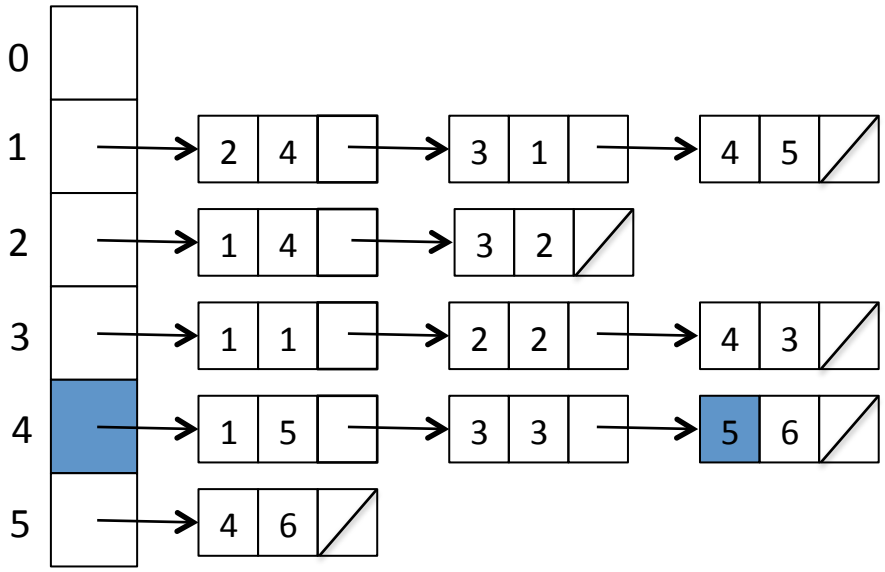
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((distance[v]+weight < distance[w])) {
         distance[w] = distance[v]+weight ;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	3	3
3	T	1	1
4	T	4	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	∞
1	1	4	3
4	3	2	
	1	5	
	3	3	
	5	6	

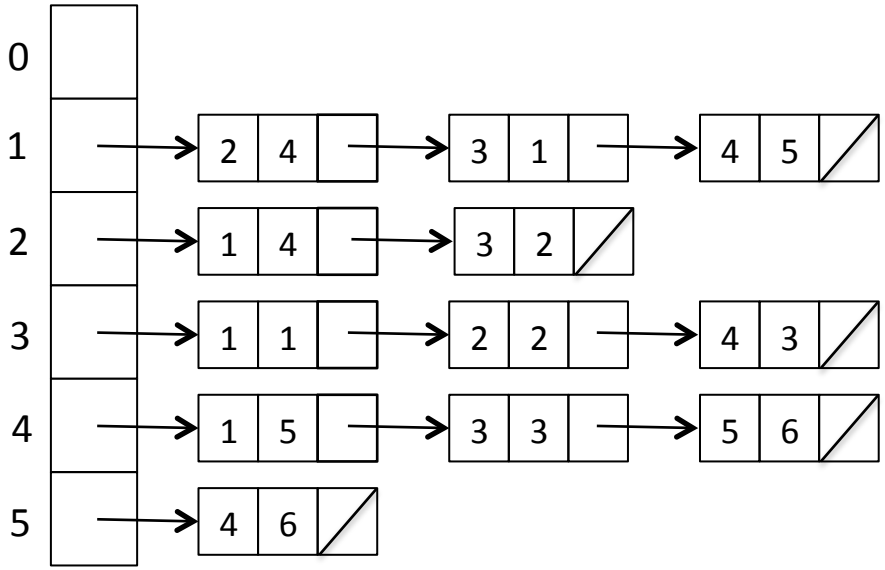
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((distance[v]+weight < distance[w])) {
         distance[w] = distance[v]+weight ;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	3	3
3	T	1	1
4	T	4	3
5	F	10	4

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	∞
1	1	4	3
4	3	2	
	1	5	
	3	3	
	5	6	

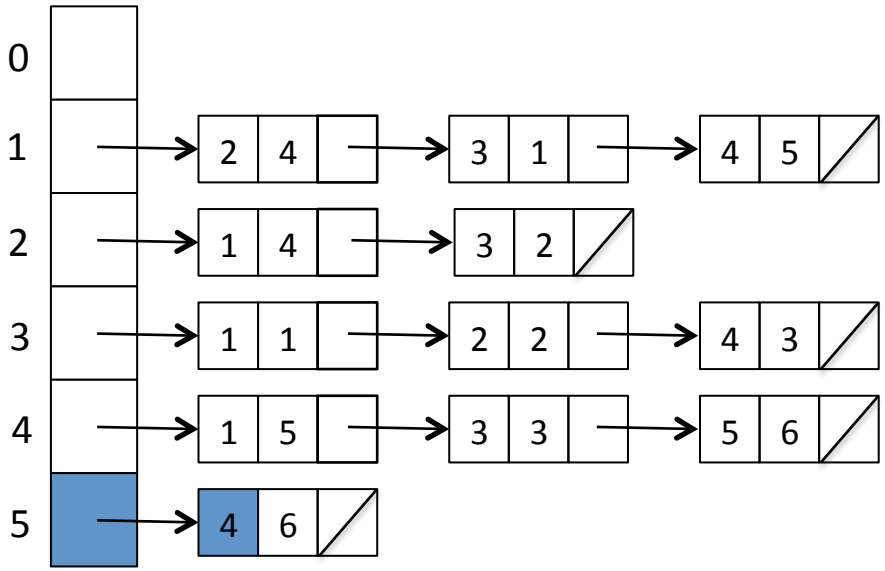
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((distance[v]+weight < distance[w])) {
         distance[w] = distance[v]+weight ;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	3	3
3	T	1	1
4	T	4	3
5	F	10	4

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	∞
1	1	4	3
4	3	2	∞
1	1	5	6
5	3	3	
		5	6

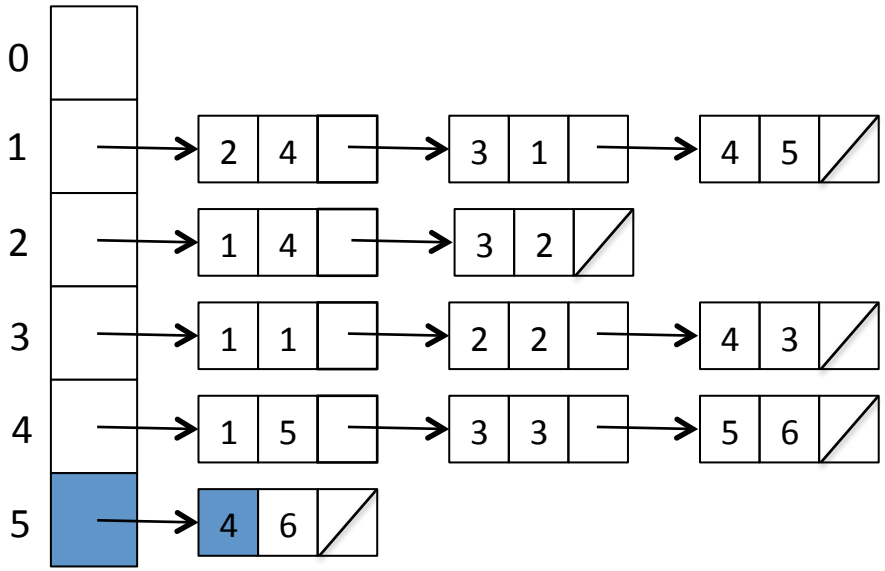
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((distance[v]+weight < distance[w])) {
         distance[w] = distance[v]+weight ;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	3	3
3	T	1	1
4	T	4	3
5	T	10	4

v	w	weight	dist
1	...	...	∞
1	4	6	4
2			1
3			∞
1			2
2			∞
1			3
4			∞
1			6
5			

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((distance[v]+weight < distance[w])) {
         distance[w] = distance[v]+weight ;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



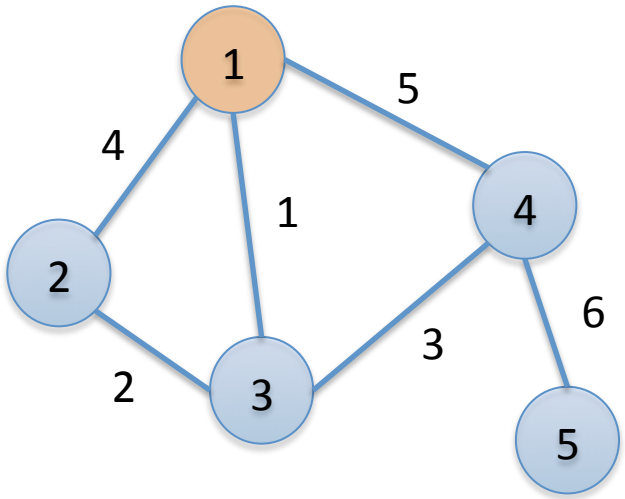
	intree	distance	parent
0			
1	T	0	-1
2	T	3	3
3	T	1	1
4	T	4	3
5	T	10	4

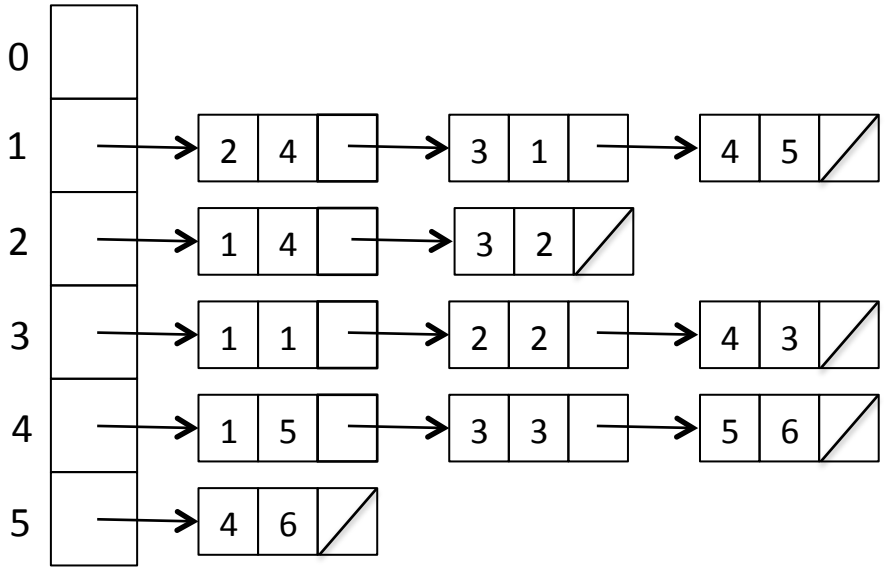
v	w	weight	dist
1	…	…	∞
1	4	6	4
2			1
3			∞
1			2
2			∞
1			3
4			∞
1			6
5			∞
1			

Shortest Paths

- Dijkstra's Algorithm
  - This implementation finds the shortest path spanning tree, i.e. shortest path between a `start` vertex and all other vertices
  - The length of the shortest path from `start` to a given vertex `t` is exactly the value of `distance[t]`
  - To find the actual path, follow the `parent` relations from `t` until we hit `start` (or -1 if no such path exists)
  - We did this in Breadth-First Search

```
find_path(int start, int end, int parents[])
```





	parent	distance
0		
1	-1	0
2	3	3
3	1	1
4	3	4
5	4	10

Shortest Paths

- All-Pairs Shortest Path – Floyd's Algorithm
  - Find the centre vertex in a graph
    - Minimize **longest or average distance to all other nodes**
    - e.g. good place to set up a pizza shop

Shortest Paths

- All-Pairs Shortest Path – Floyd's Algorithm
  - Find the diameter of a graph
    - Minimize **longest shortest-path distance over all pairs of vertices**
    - e.g. longest possible time for a network packet to be delivered
  - Both examples require computation of the **shortest path between all pairs of vertices** in a given graph (n x n distance matrix)

Shortest Paths

- All-Pairs Shortest Path – Floyd's Algorithm
  - Simple solution: call Dijkstra's algorithm from each of the n possible starting vertices (hence n x n distance matrix)
  - Floyd-Warshall Algorithm
    - Construct n x n shortest-path distance matrix directly from original n x n weight matrix
    - Use adjacency matrix instead of adjacency list data structure

Shortest Paths

- All-Pairs Shortest Path – Floyd's Algorithm

Use adjacency matrix instead of adjacency list data structure

```
typedef struct {
  int weight[MAXV+1][MAXV+1];  /* adjacency/weight info       */
  int nvertices;               /* number of vertices in graph */
} adjacency_matrix;
```

Initialize each non-edge to `MAXINT (INT_MAX)` instead of zero

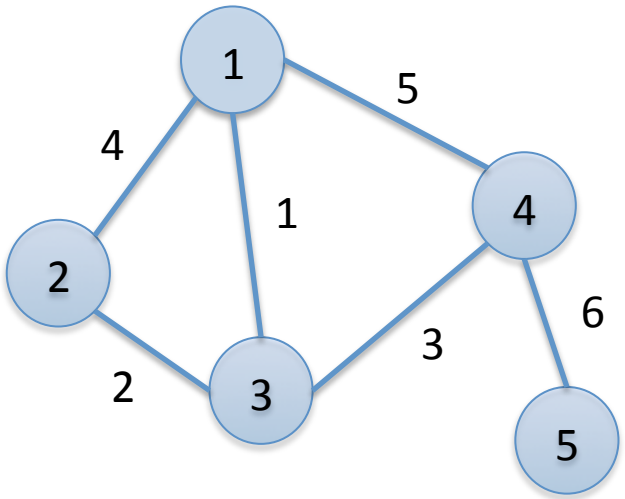
Will flag non-edge and still be ignored in shortest path algorithm

Shortest Paths

- Floyd-Warshall Algorithm
  - Number all the vertices from 1 to *n*
    - Use these numbers to order the vertices (not label them)
  - Define $W[i, j]^k$ to be the length of the shortest path from *i* to j using vertices numbered from $1, 2, \ldots, k$ as possible intermediate vertices

Shortest Paths

- Floyd-Warshall Algorithm



∞	4	1	5	∞
4	∞	2	∞	∞
1	2	∞	3	∞
5	∞	3	∞	6
∞	∞	∞	6	∞

Shortest Paths

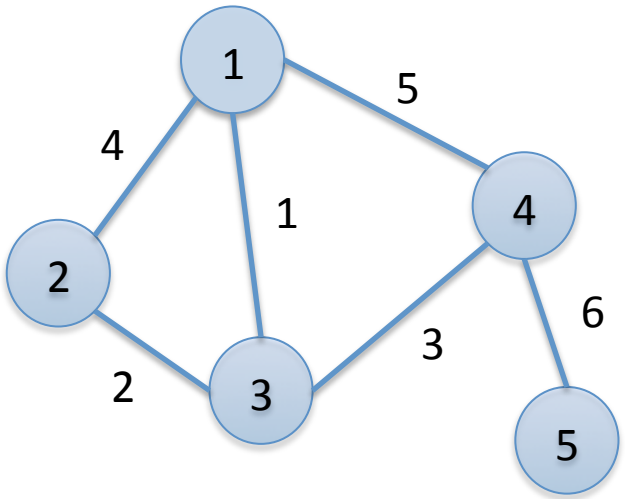
- Floyd-Warshall Algorithm
  - Perform k iterations
  - Iteration k allows only the first k vertices as possible intermediate steps on the path from between each pair of vertices x and y
  - With each iteration we allow a richer set of possible shortest paths by adding a new vertex as a possible intermediary
  - Allowing the kth vertex as an intermediary stop helps only if there is a short path that goes through k

$$W[i, j]^k = \min(W[i, j]^{k-1}, W[i, k]^{k-1} + W[k, j]^{k-1}$$

This about this …

Shortest Paths

- Floyd-Warshall Algorithm



∞	4	1	5	∞
4	8	2	9	∞
1	2	2	3	∞
5	9	3	10	6
∞	∞	∞	6	∞

$$W[i, j]^k = \min(W[i, j]^{k-1}, W[i, k]^{k-1} + W[k, j]^{k-1} \quad k=1$$

Shortest Paths

```
floyd(adjacency_matrix *g) {
   int i,j;        /* dimension counters          */
   int k;          /* intermediate vertex counter */
   int through_k; /* distance through vertex k    */

   for (k=1; k<=g->nvertices; k++) {
      for (i=1; i<=g->nvertices; i++) {
         for (j=1; j<=g->nvertices; j++) {
            through_k = g->weight[i][k]+g->weight[k][j];
            if (through_k < g->weight[i][j]) {
                g->weight[i][j] = through_k;
            }
         }
      }
   }
}
```

Shortest Paths

- Floyd-Warshall Algorithm
  - $O(n^3)$
  - No better than n calls to Dijkstra's algorithm
  - Better in practice (tight loops)
  - One of the few algorithms that work better on adjacency matrices
  - Does not allow you to construct the actual shortest path between any given pair of vertices (not the point of the algorithm)