04-630
Data Structures and Algorithms for Engineers

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Lecture 21

Graphs

- Types of graph
- Adjacency matrix representation
- Adjacency list representation
- Breadth-First Search traversal
- Depth-First Search traversal
- Topological Sorting
- Minimum Spanning Tree
  - Prim's algorithm
  - Kruskal's algorithm
- Shortest Path Algorithms
  - Dijkstra's algorithm
  - Floyd's algorithm

Minimum Spanning Tree

- A spanning tree of a graph $G = (V, E)$ is a subset of edges from E forming a tree connecting all vertices of V

- For edge-weighted graphs we are interested in the minimum spanning tree*:*

  – The tree whose sum of edge weights is as small as possible

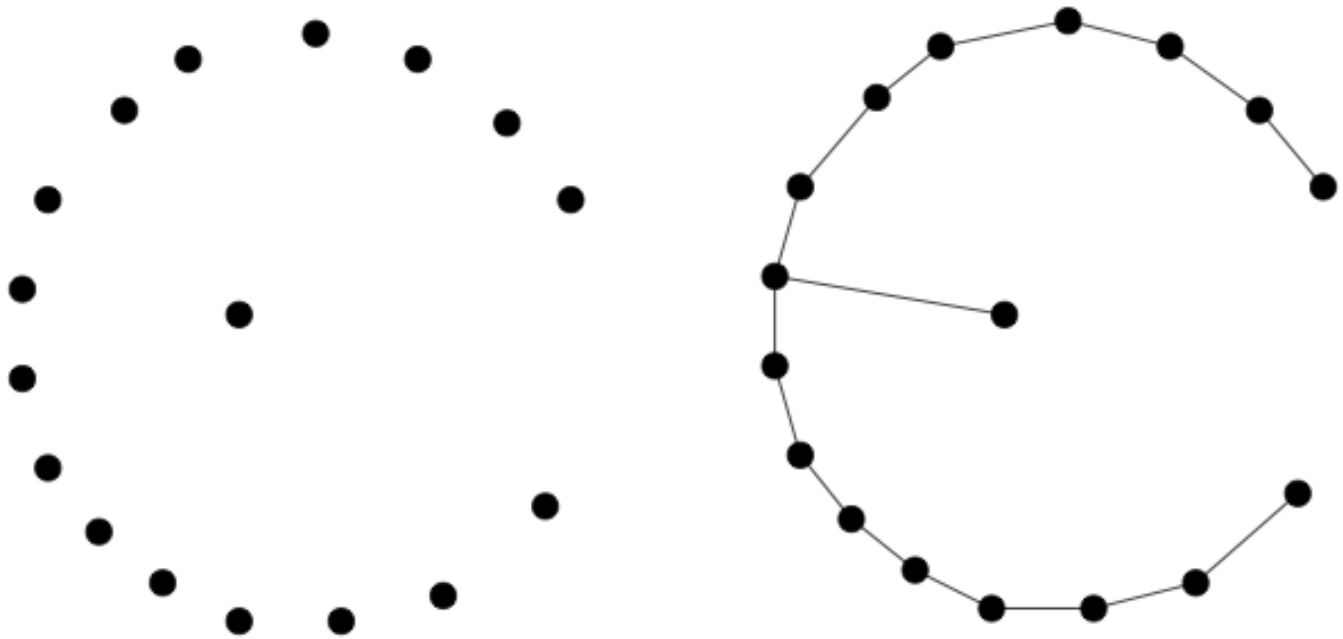
Minimum Spanning Tree

Example application:

- find the smallest amount of a connector (road, wire, pipe)

  required to connect a set of points (cities, houses, components)

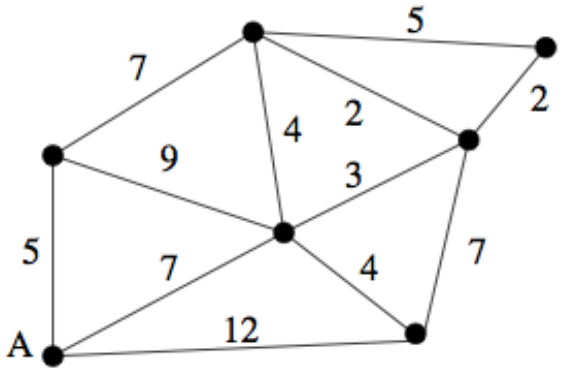
Minimum Spanning Tree



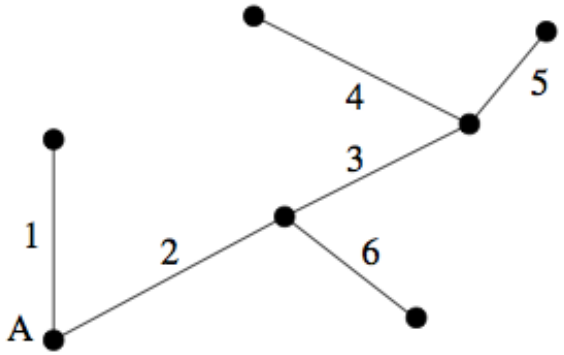
Minimum Spanning Tree

- A minimum spanning tree minimizes the total length (weight) over all possible spanning trees
- There can be more than one minimum spanning tree in a graph
- Two main algorithms
  - Prim's Algorithm
  - Kruskal's Algorithm

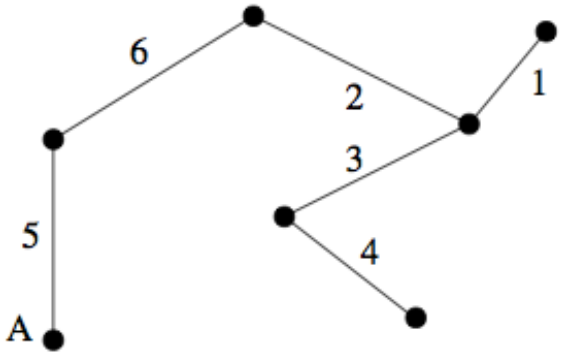
Minimum Spanning Tree



G



Prim(G,A)



Kruskal(G)

Numbers denote order of insertion

Minimum Spanning Tree

Prim's Algorithm

- Greedy algorithmic strategy:

  Make a decision about what to do next by selecting the best local option from all available choices without regard to global structure,

  do it,

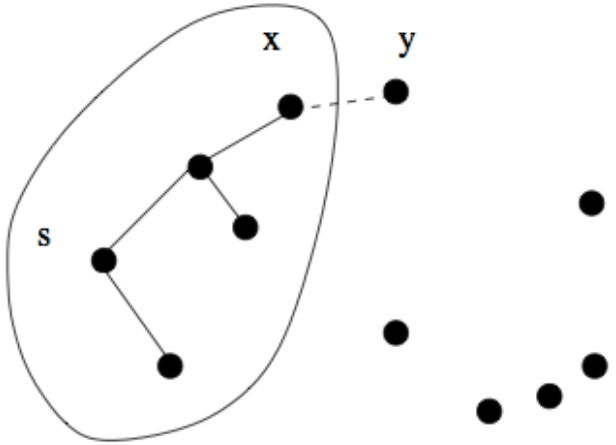
  and then repeat until finished

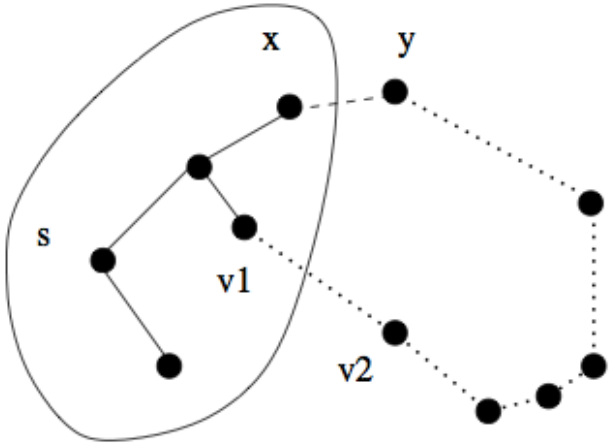
Minimum Spanning Tree

Prim's Algorithm

– Repeatedly select the smallest weight edge that will enlarge the number of vertices in the tree

- Start from one (any) vertex,
- Grow the rest of the tree, one edge at a time,
- Until all vertices are included





Minimum Spanning Tree

Prim's Algorithm

Prim-MST(G):

Select an arbitrary vertex s to start the tree from

While (there are still non-tree vertices remaining)

Select the edge of minimum weight between a tree & non-tree vertex

Add the selected edge and vertex to the tree T_prim

Minimum Spanning Tree

```
/* Prim's algorithm                                                    */

prim(graph *g, int start) {
   int i;                  /* counter                                 */
   edgenode *p;            /* temporary pointer                       */
   bool intree[MAXV+1];    /* is the vertex in the tree yet?          */
   int distance[MAXV+1];   /* cost of adding to tree                  */
   int parent[MAXV+1];     /* parent vertex                           */
   int v;                  /* current vertex to process               */
   int w;                  /* candidate next vertex                   */
   int weight;             /* edge weight                             */
   int dist;               /* best current distance from start        */

   for (i=1; i<=g->nvertices; i++) {
      intree[i] = FALSE;
      distance[i] = MAXINT;
      parent[i] = -1;
   }

   distance[start] = 0;
   v = start;
```

Minimum Spanning Tree

```
    while (intree[v] == FALSE) { // keep going until all vertices
       intree[v] = TRUE;         // are in the tree
       p = g->edges[v];
       while (p != NULL) {       // compute the distances and parents
          w = p->y;
          weight = p->weight;
          if ((weight < distance[w]) && (intree[w] == FALSE)) {
             distance[w] = weight;
             parent[w] = v;
          }
          p = p->next;
       }
       v = 1;
       dist = MAXINT;                    // find the vertex with
       for (i=1; i<=g->nvertices; i++)   // the minimum distance
          if ((intree[i] == FALSE) && (distance[i] < dist)) {
             dist = distance[i];
             v = i;
          }
    }
}
```

Minimum Spanning Tree

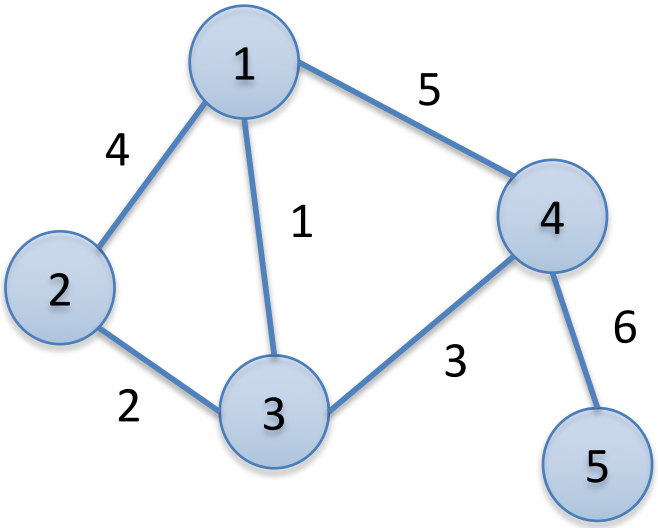
```
while (intree[v] == FALSE) { // keep going until all vertices
   intree[v] = TRUE;         // are in the tree
   p = g->edges[v];
   while (p != NULL) {       // compute the distances and parents
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&(intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }
   v = 1;
   dist = MAXINT;                        // find the vertex with
   for (i=1; i<=g->nvertices; i++)       // the minimum distance
      if ((intree[i] == FALSE) && (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
   }
}
```

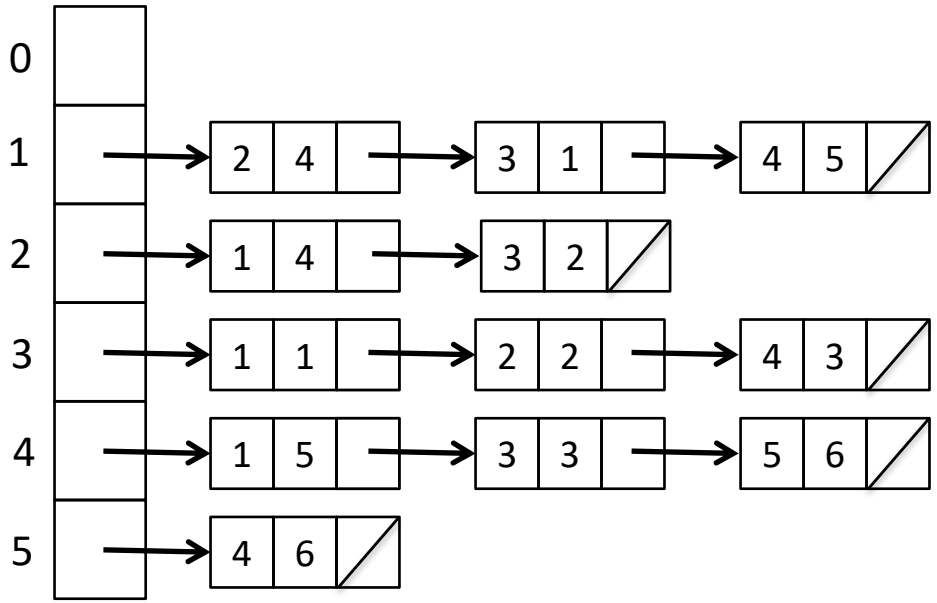
Find the weights of all edges from v

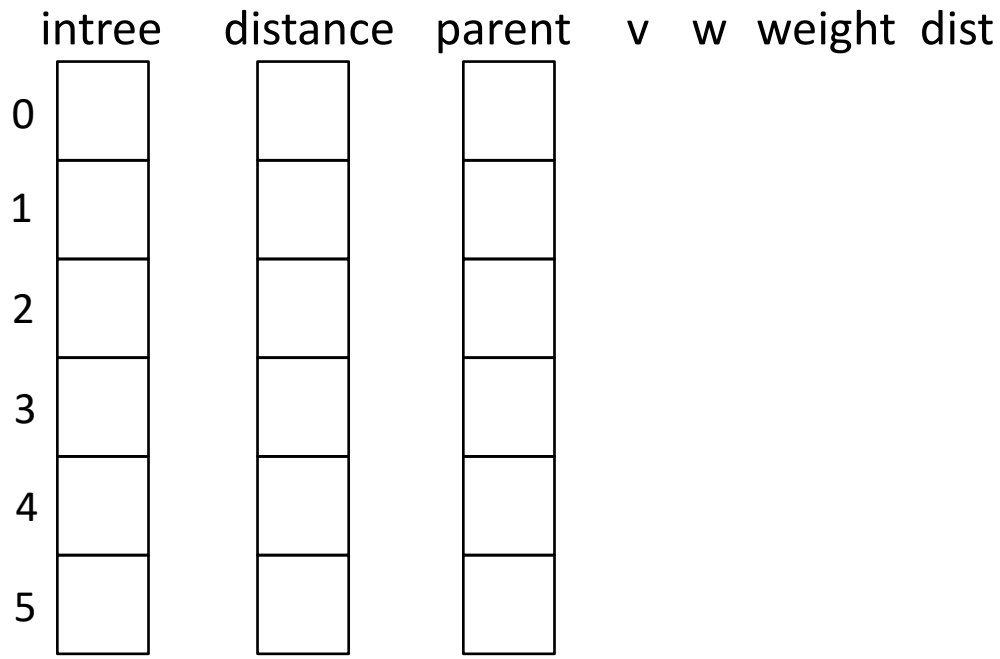
Minimum Spanning Tree

```
    while (intree[v] == FALSE) { // keep going until all vertices
       intree[v] = TRUE;         // are in the tree
       p = g->edges[v];
       while (p != NULL) {       // compute the distances and parents
          w = p->y;
          weight = p->weight;
          if ((weight < distance[w]) && (intree[w] == FALSE)) {
             distance[w] = weight;
             parent[w] = v;
          }
          p = p->next;
       }
       v = 1;
       dist = MAXINT;                    // find the vertex with
       for (i=1; i<=g->nvertices; i++)   // the minimum distance
          if ((intree[i] == FALSE) && (distance[i] < dist)) {
             dist = distance[i];
             v = i;
          }
    }
}
```

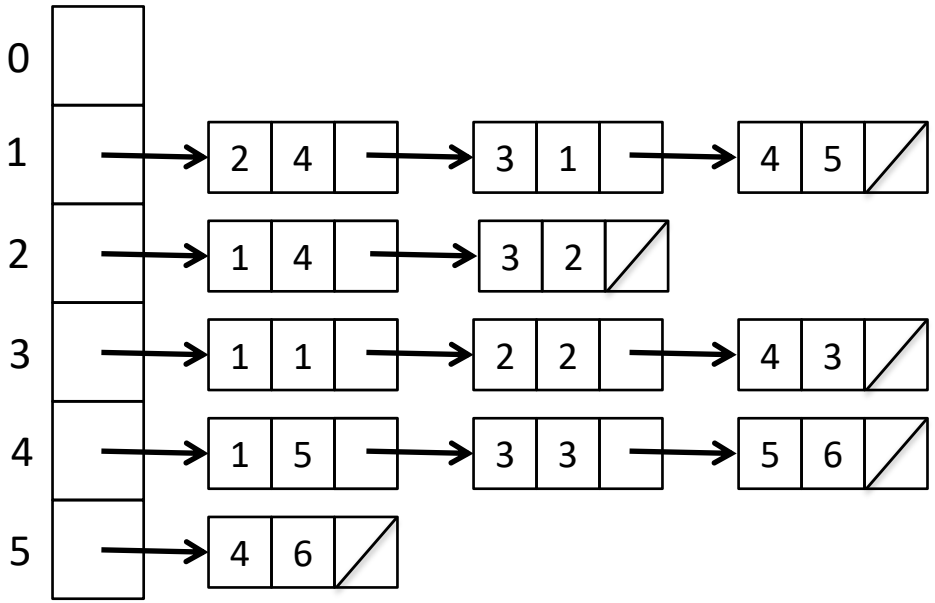
Find the minumum weight … this gives the vertex to be added to the MST

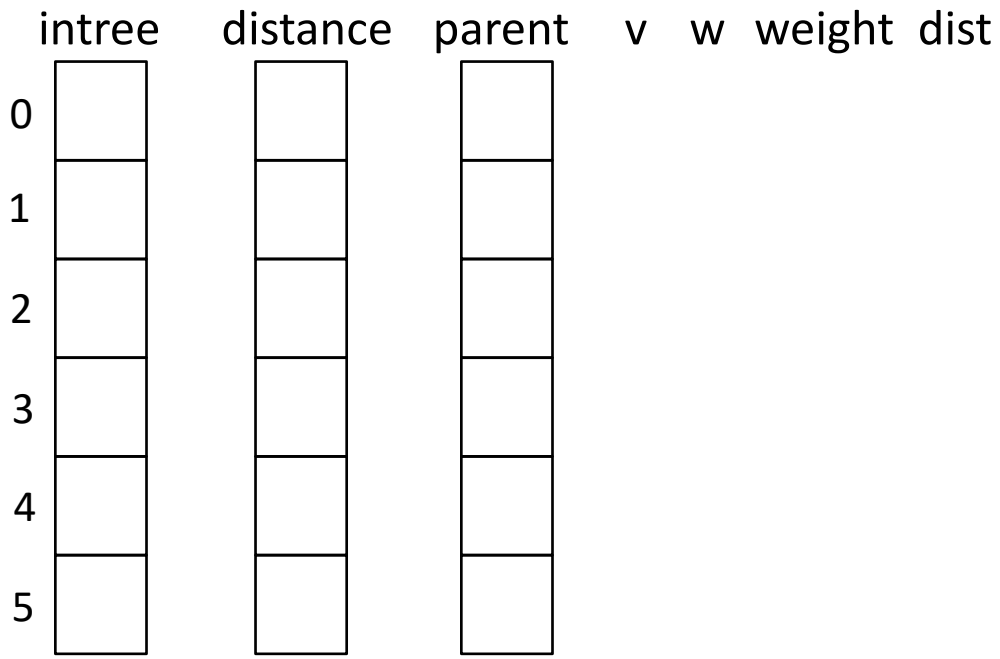

1
2
3
4
5
5
4
1
6
3
2


0
1
2
3
4
5
2 4
3 1
4 5
1 4
3 2
1 1
2 2
4 3
1 5
3 3
5 6
4 6


intree
distance
parent
v
w
weight
dist
0
1
2
3
4
5

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```





```
for (i=1; i<=g->nvertices; i++) {
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         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```

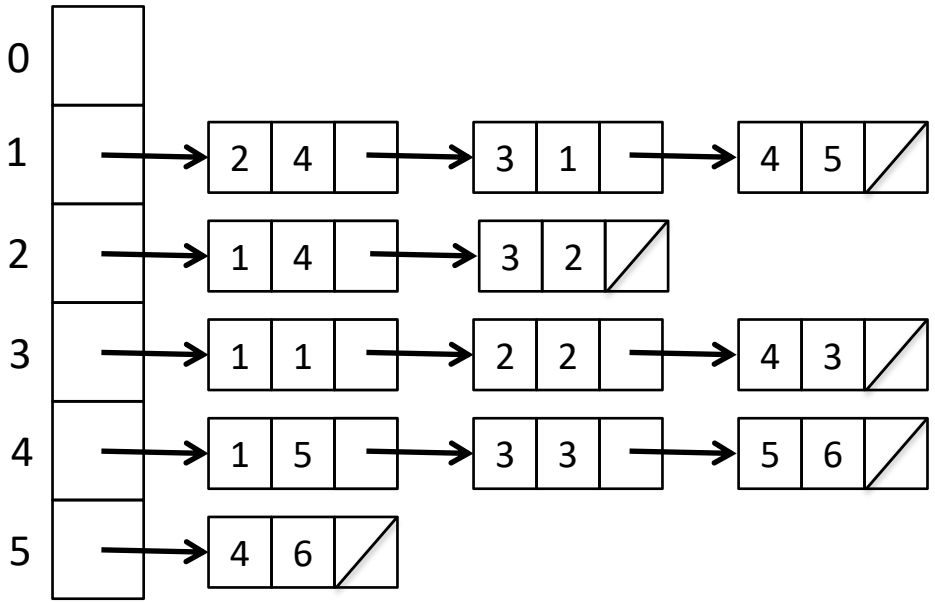

	intree	distance	parent
0			
1	F	∞	-1
2	F	∞	-1
3	F	∞	-1
4	F	∞	-1
5	F	∞	-1

v w weight dist

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
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   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```

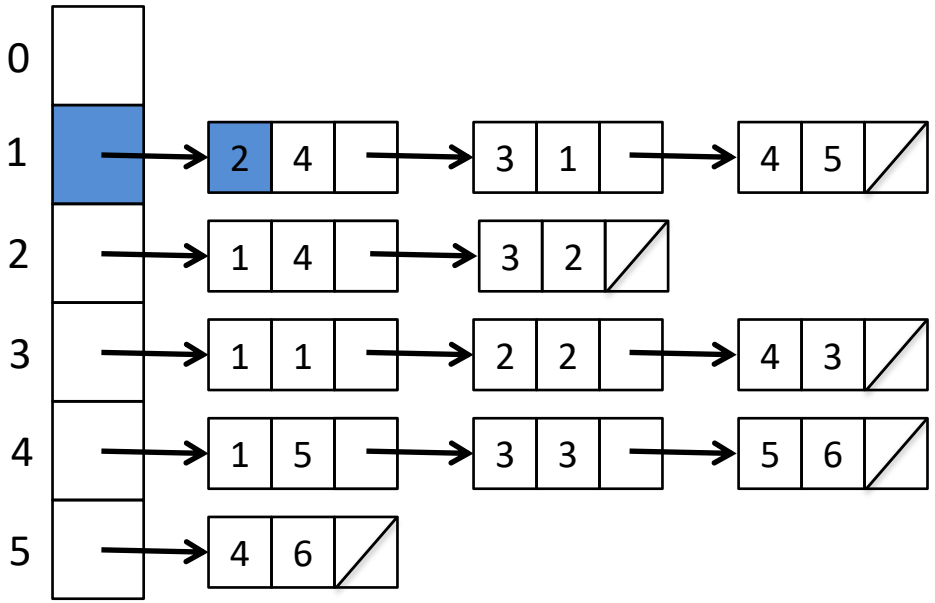

	intree	distance	parent
0			
1	T	0	-1
2	F	∞	-1
3	F	∞	-1
4	F	∞	-1
5	F	∞	-1

v	w	weight	dist
1	2	4	

```
for (i=1; i<=g->nvertices; i++) {
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}

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         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```

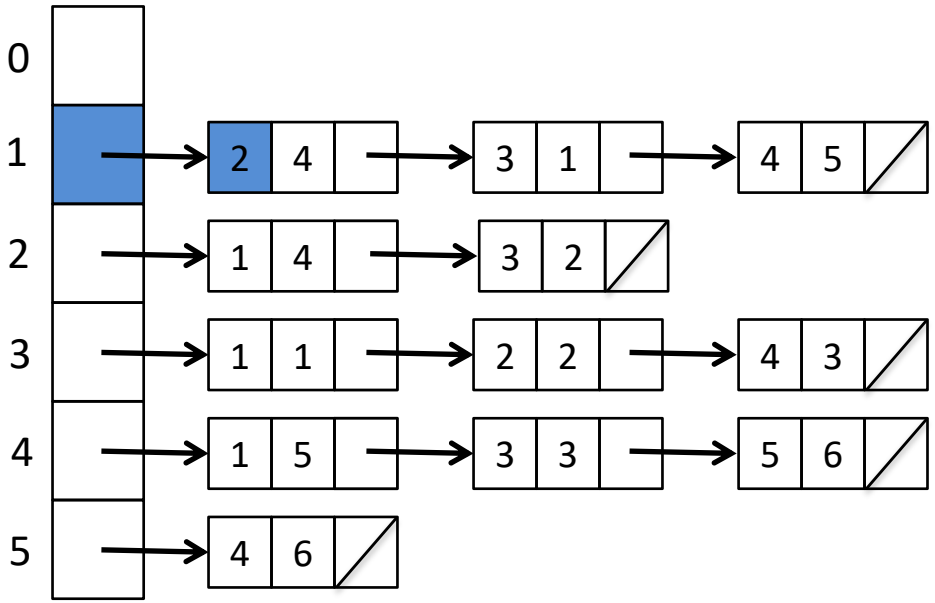

	intree	distance	parent
0			
1	T	0	-1
2	F	4	1
3	F	∞	-1
4	F	∞	-1
5	F	∞	-1

v	w	weight	dist
1	2	4	

```
for (i=1; i<=g->nvertices; i++) {
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         parent[w] = v;
      }
      p = p->next;
   }

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   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```

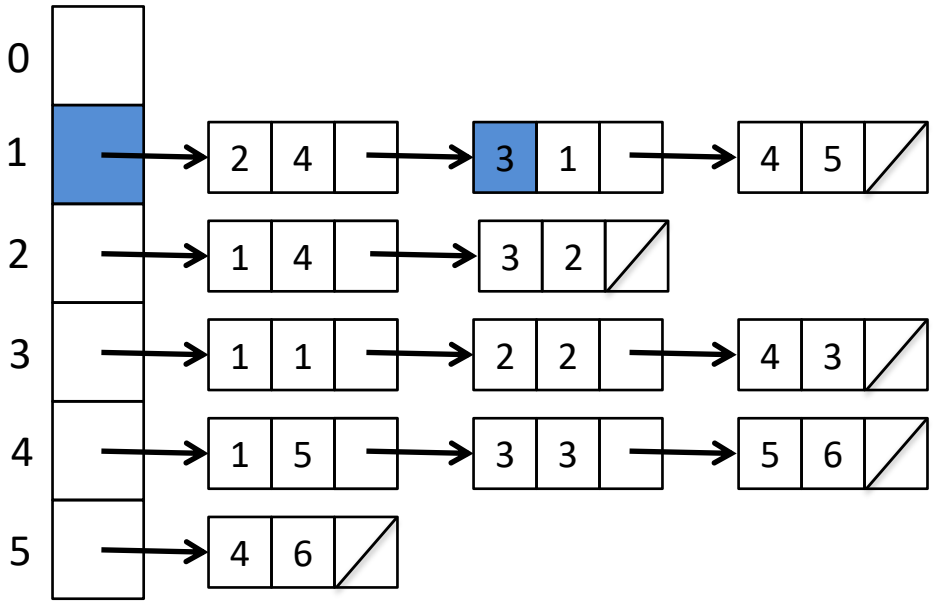

	intree	distance	parent
0			
1	T	0	-1
2	F	4	1
3	F	1	1
4	F	∞	-1
5	F	∞	-1

v	w	weight	dist
1	2	4	
	3	1	

```
for (i=1; i<=g->nvertices; i++) {
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      }
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   }

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   dist = MAXINT;
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          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```

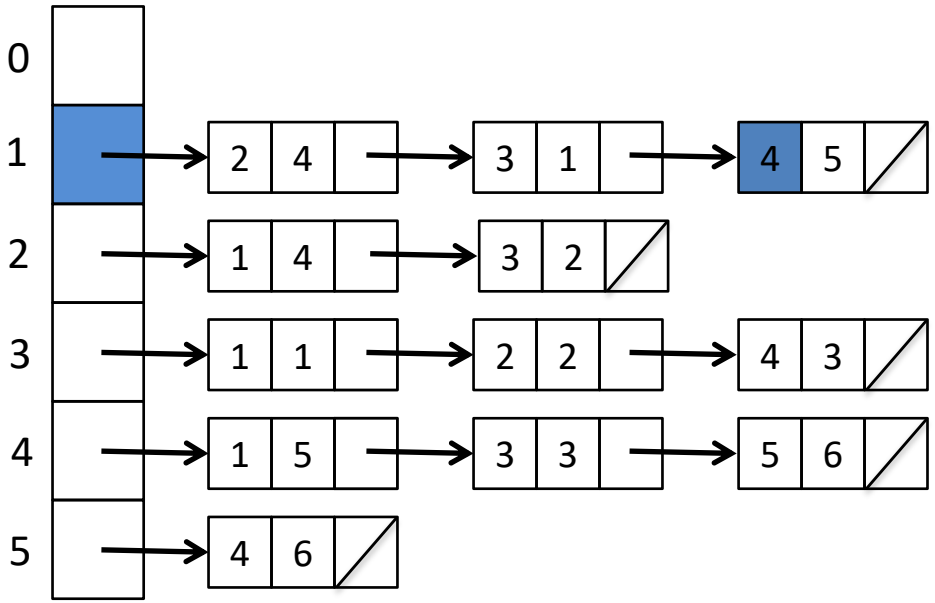

	intree	distance	parent
0			
1	T	0	-1
2	F	4	1
3	F	1	1
4	F	5	1
5	F	∞	-1

v	w	weight	dist
1	2	4	
	3	1	
	4	5	

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
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}

distance[start] = 0;
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          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```

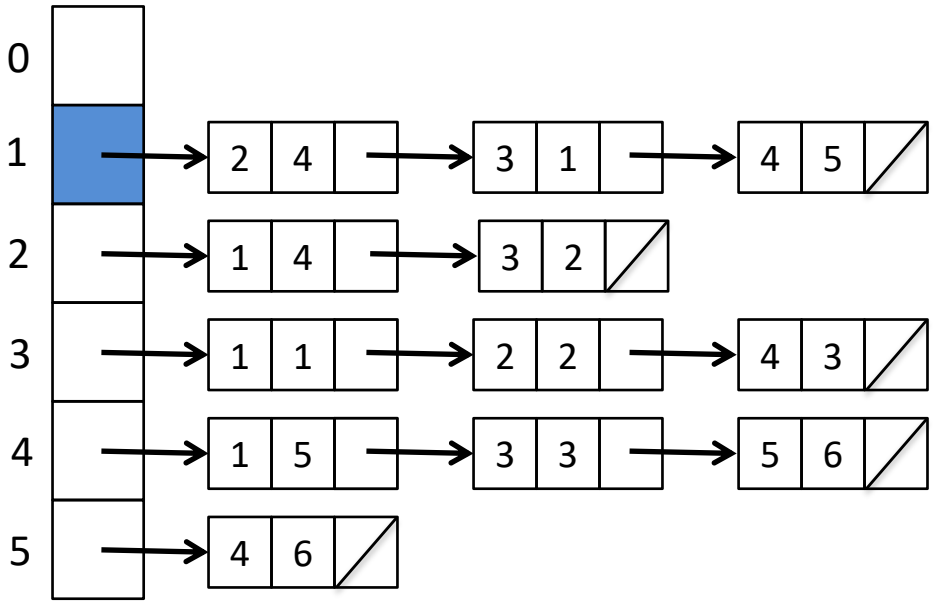

	intree	distance	parent
0			
1	T	0	-1
2	F	4	1
3	F	1	1
4	F	5	1
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3			

```
for (i=1; i<=g->nvertices; i++) {
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   intree[v] = TRUE;
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         distance[w] = weight;
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      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```

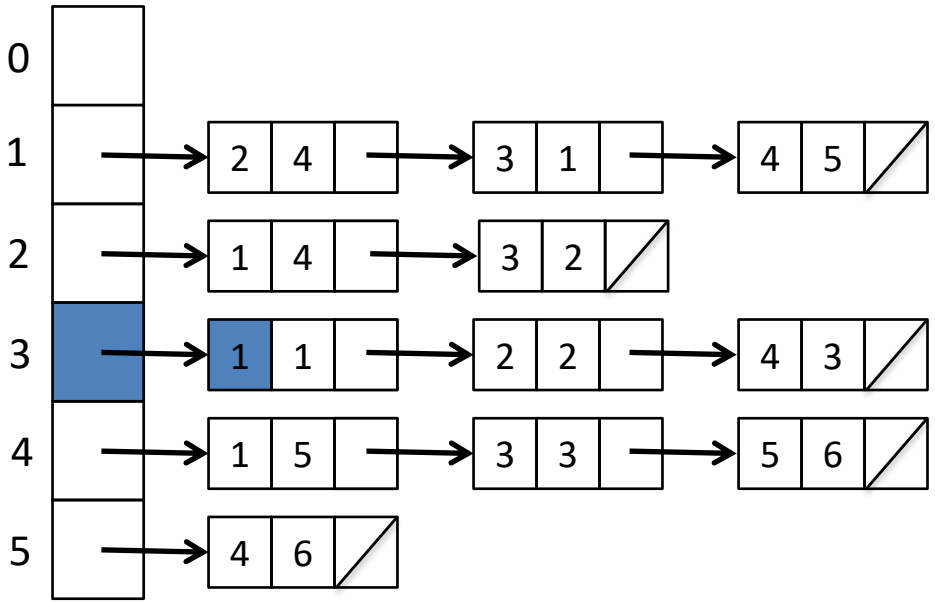

	intree	distance	parent
0			
1	T	0	-1
2	F	4	1
3	T	1	1
4	F	5	1
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	

```
for (i=1; i<=g->nvertices; i++) {
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distance[start] = 0;
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   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
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      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```

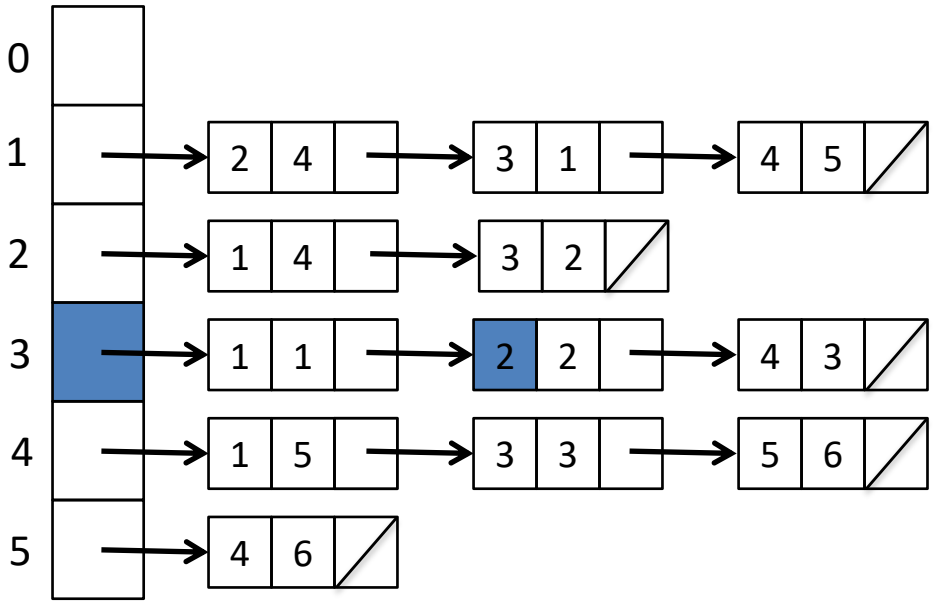

	intree	distance	parent
0			
1	T	0	-1
2	F	4	1
3	T	1	1
4	F	5	1
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	
	2	2	

```
for (i=1; i<=g->nvertices; i++) {
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   p = g->edges[v];

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      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```

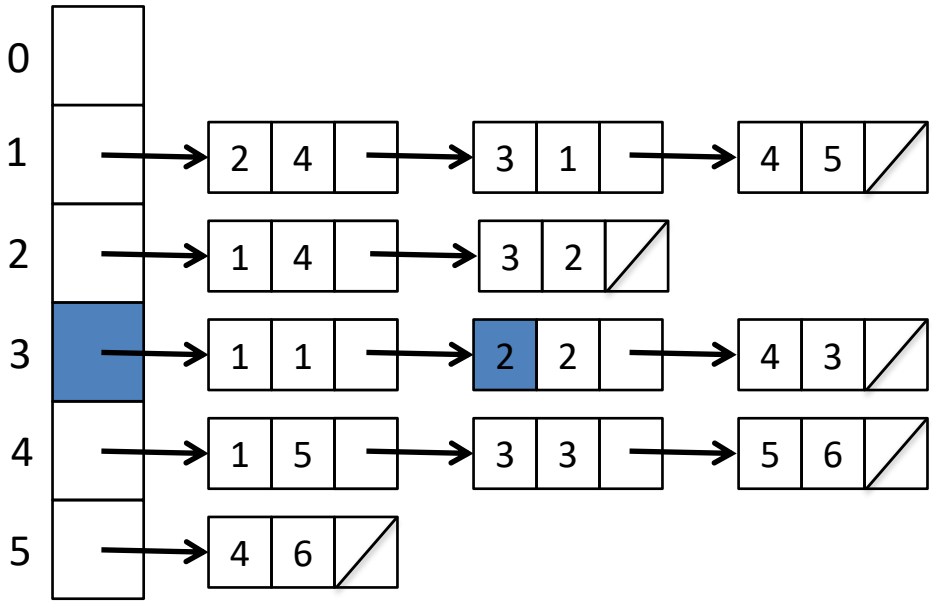

	intree	distance	parent
0			
1	T	0	-1
2	F	2	3
3	T	1	1
4	F	5	1
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	
	2	2	

```
for (i=1; i<=g->nvertices; i++) {
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          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```
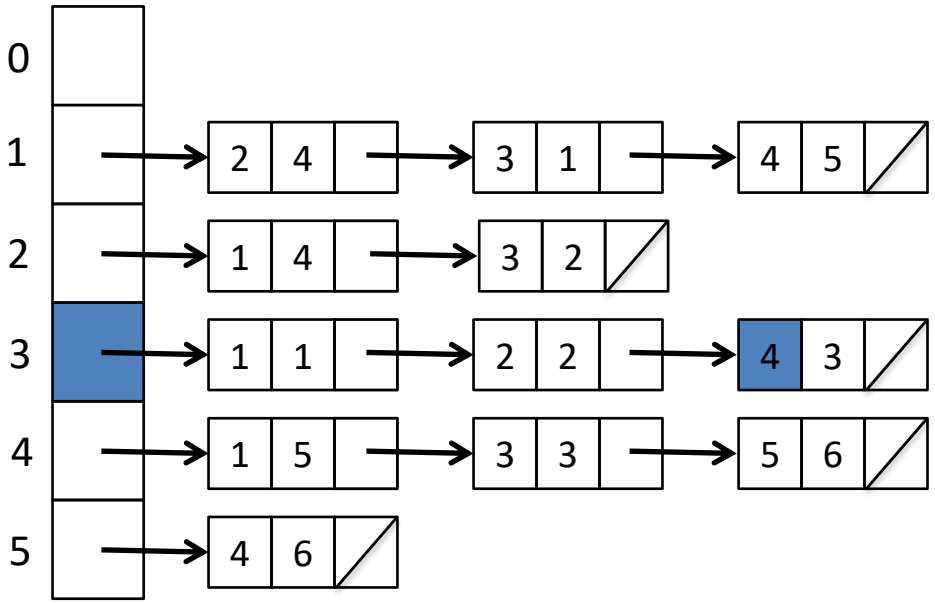
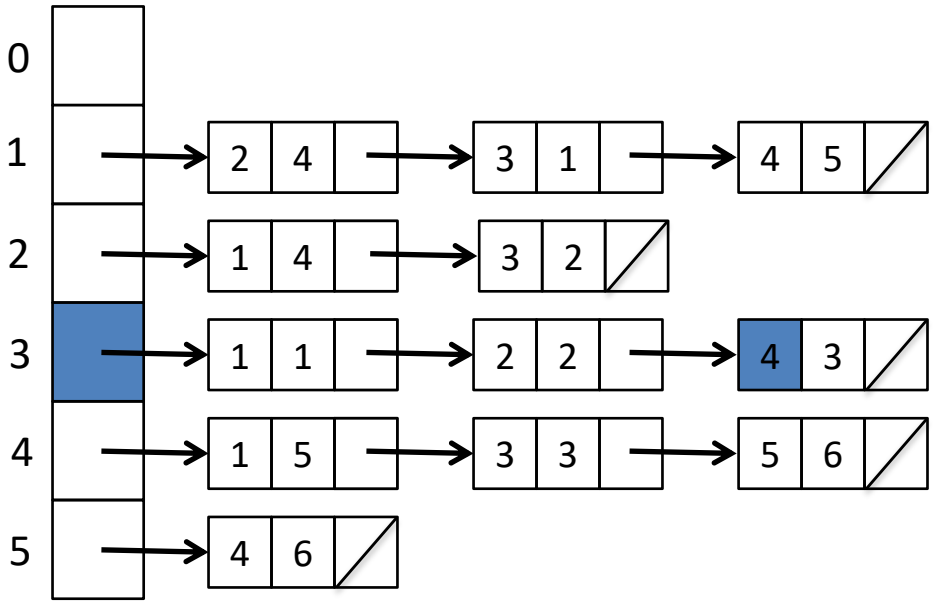



	intree	distance	parent
0			
1	T	0	-1
2	F	2	3
3	T	1	1
4	F	5	1
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	
	2	2	

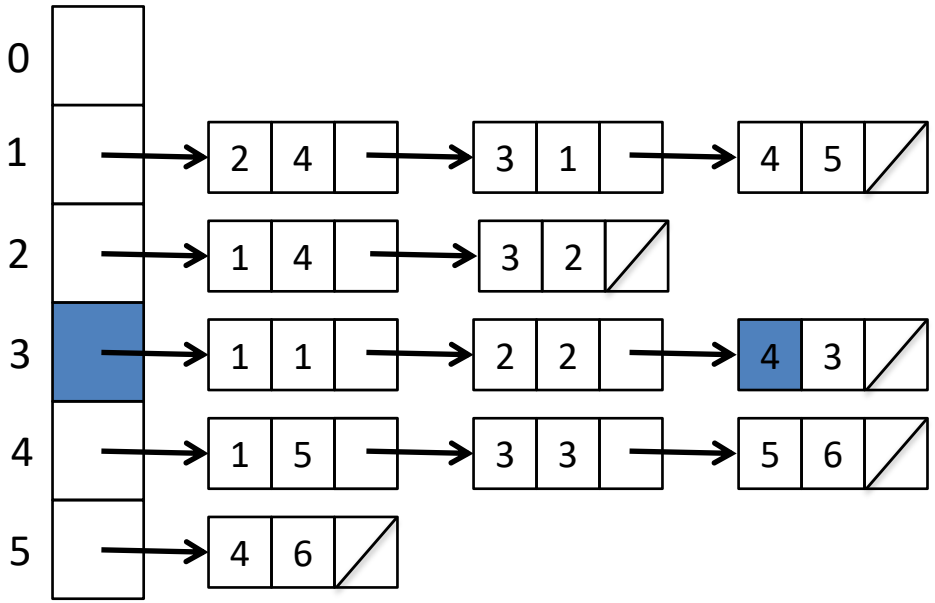
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	F	2	3
3	T	1	1
4	F	5	1
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	
	2	2	
	4	3	

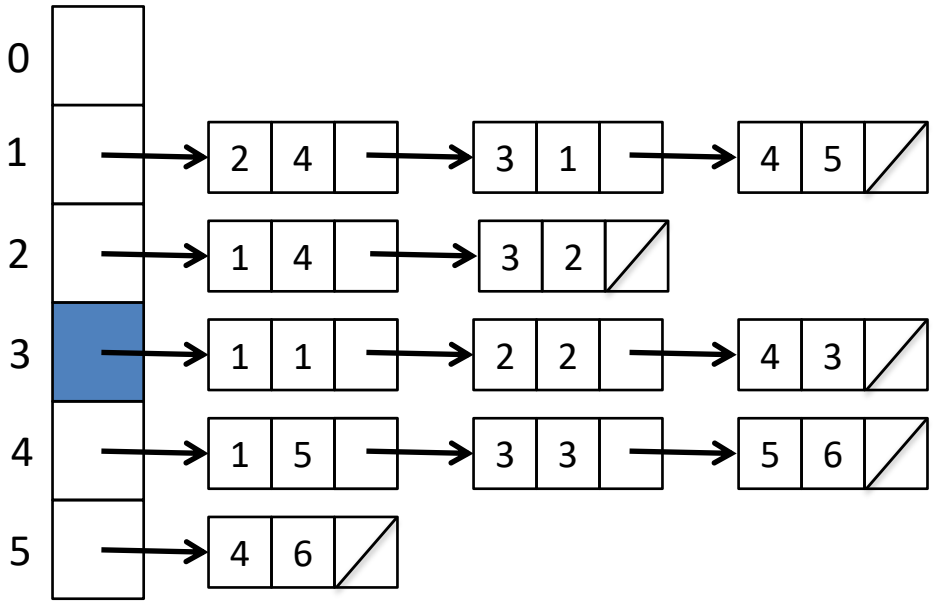
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	F	2	3
3	T	1	1
4	F	3	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	
	2	2	
	4	3	

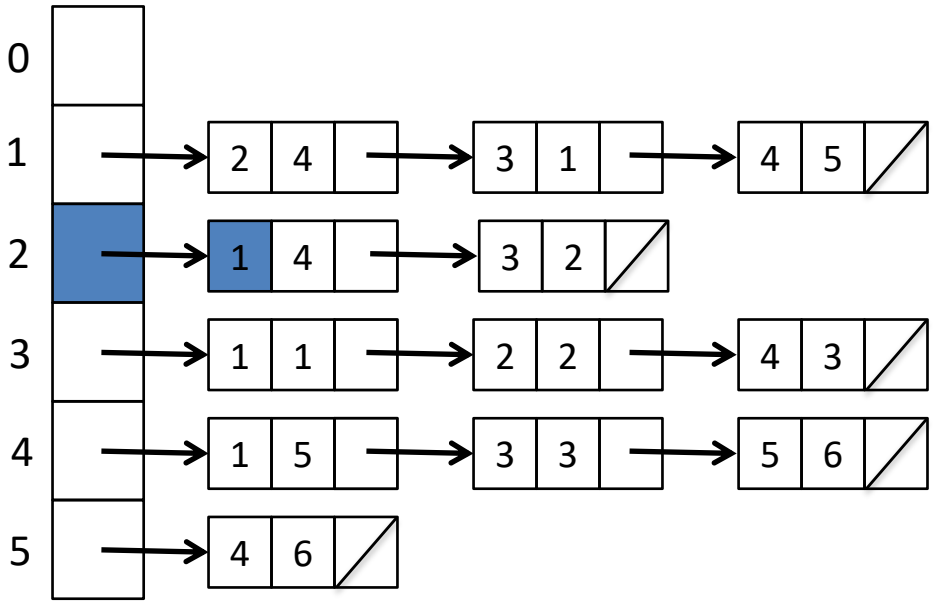
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	F	2	3
3	T	1	1
4	F	3	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	

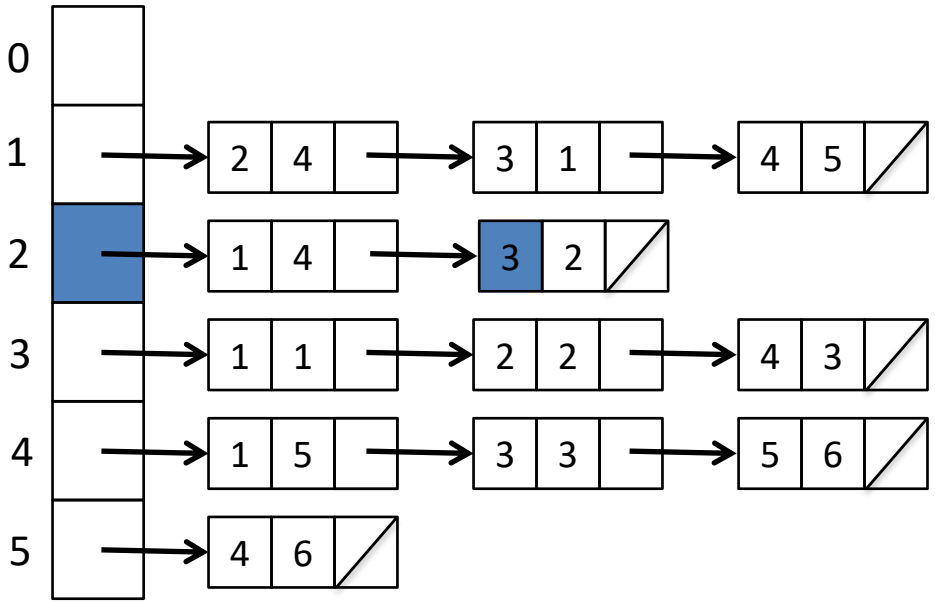
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	2	3
3	T	1	1
4	F	3	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	
	1	4	

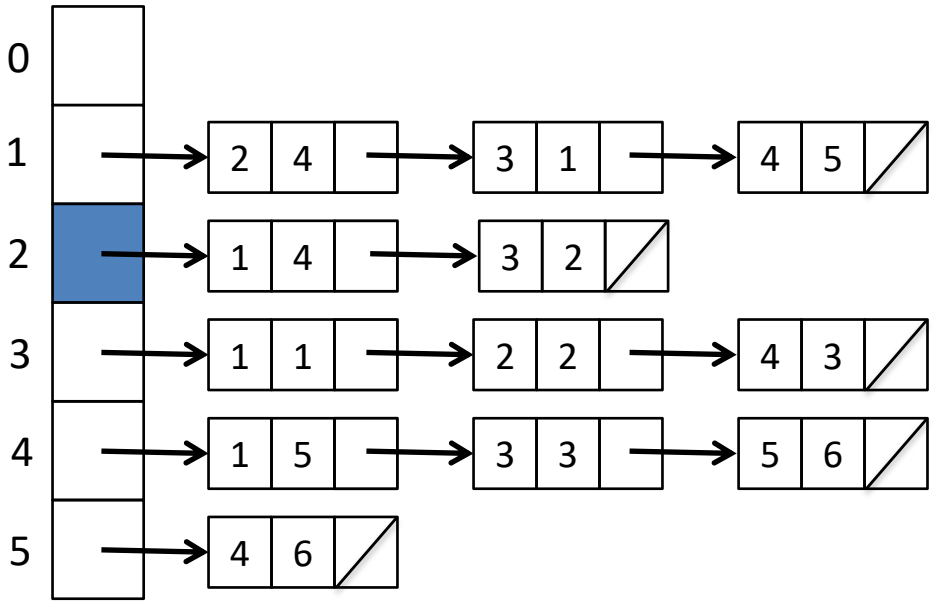
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	2	3
3	T	1	1
4	F	3	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	
	1	4	
	3	2	

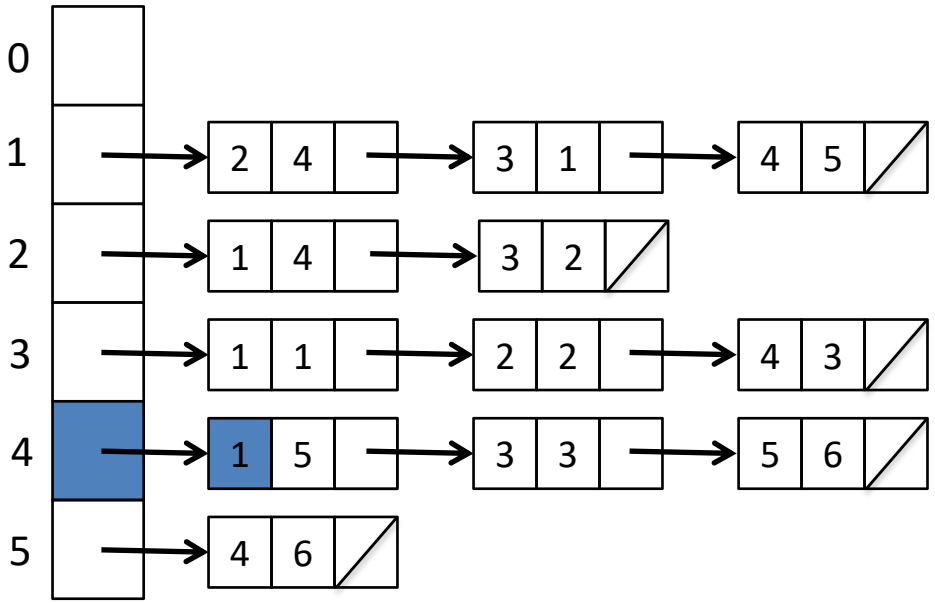
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	2	3
3	T	1	1
4	F	3	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	∞
1	1	4	3
4	3	2	

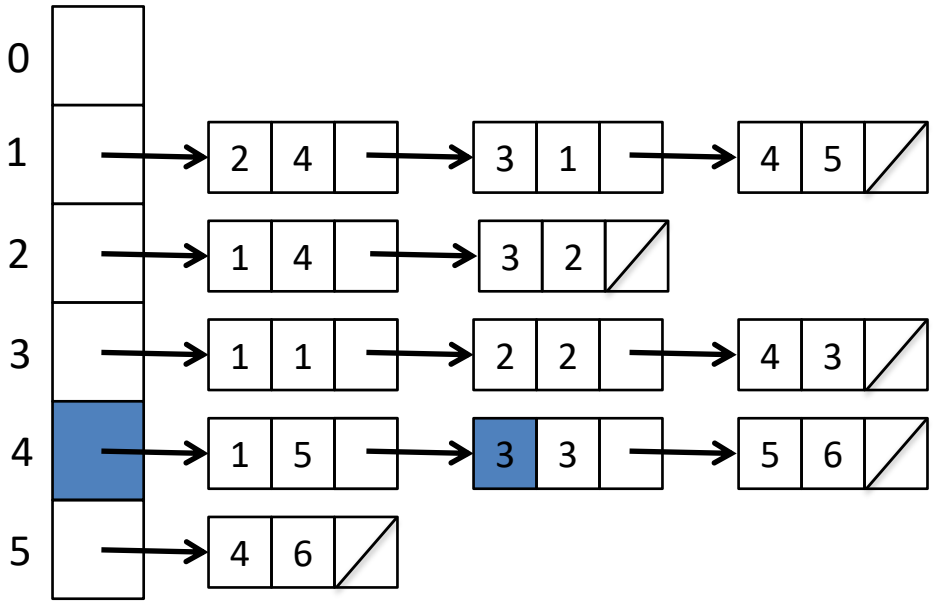
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	2	3
3	T	1	1
4	T	3	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	∞
1	1	4	3
4	3	2	
	1	5	

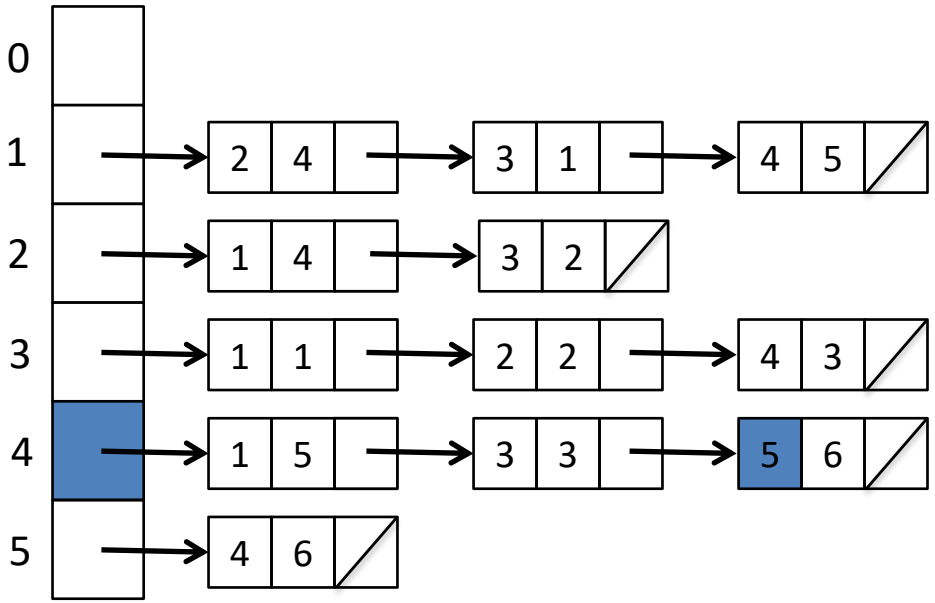
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	2	3
3	T	1	1
4	T	3	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	∞
1	1	4	3
4	3	2	
	1	5	
	3	3	

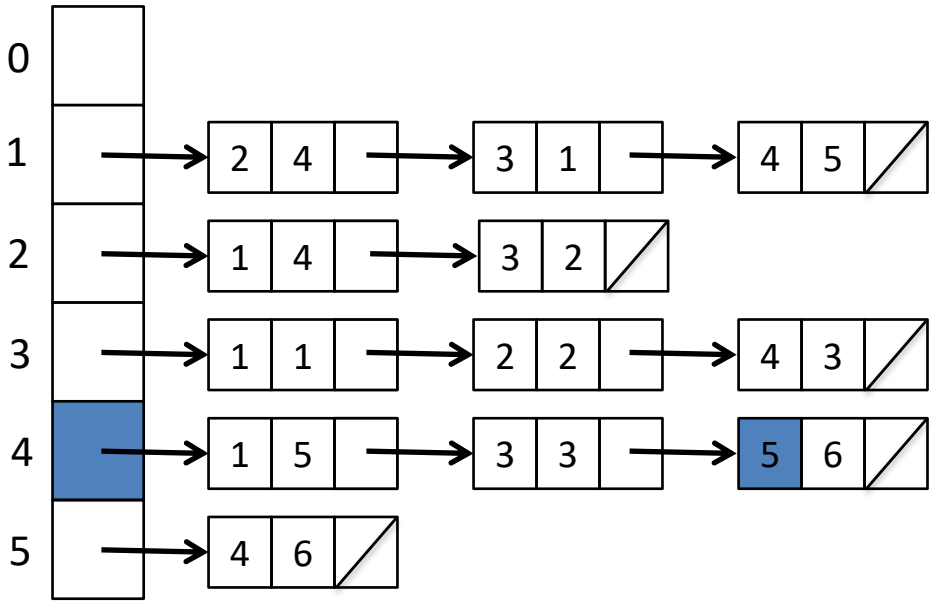
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	2	3
3	T	1	1
4	T	3	3
5	F	∞	-1

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	∞
1	1	4	3
4	3	2	
	1	5	
	3	3	
	5	6	

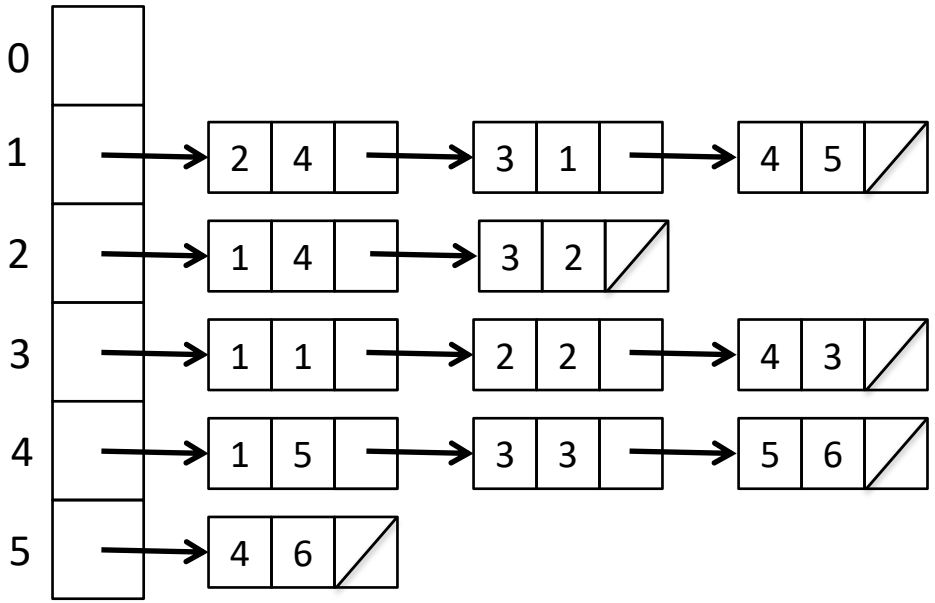
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	2	3
3	T	1	1
4	T	3	3
5	F	6	4

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	∞
1	1	4	3
4	3	2	
	1	5	
	3	3	
	5	6	

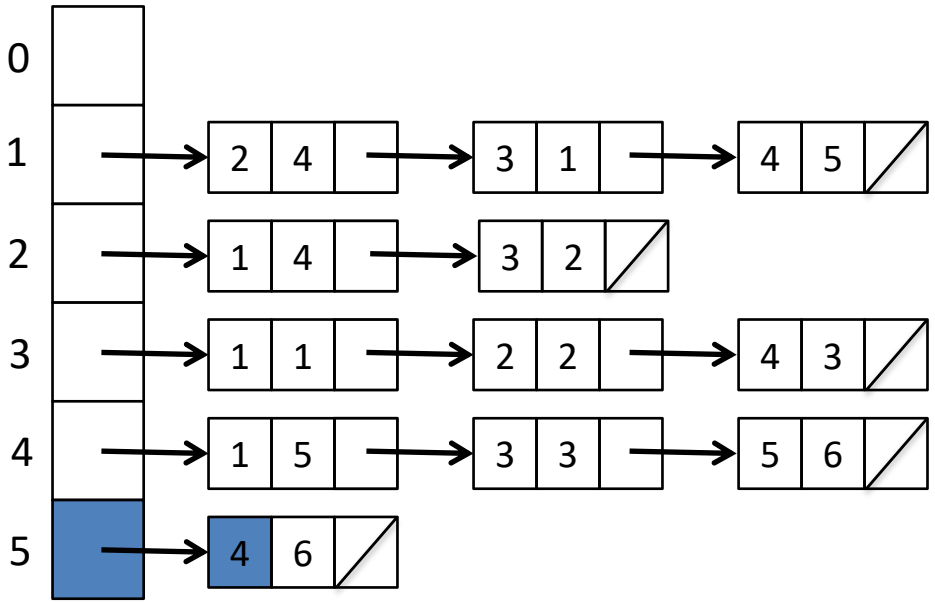
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	2	3
3	T	1	1
4	T	3	3
5	F	6	4

v	w	weight	dist
1	2	4	∞
1	3	1	4
2	4	5	1
3	1	1	∞
1	2	2	2
2	4	3	∞
1	1	4	3
4	3	2	∞
1	1	5	6
5	3	3	
5	6		

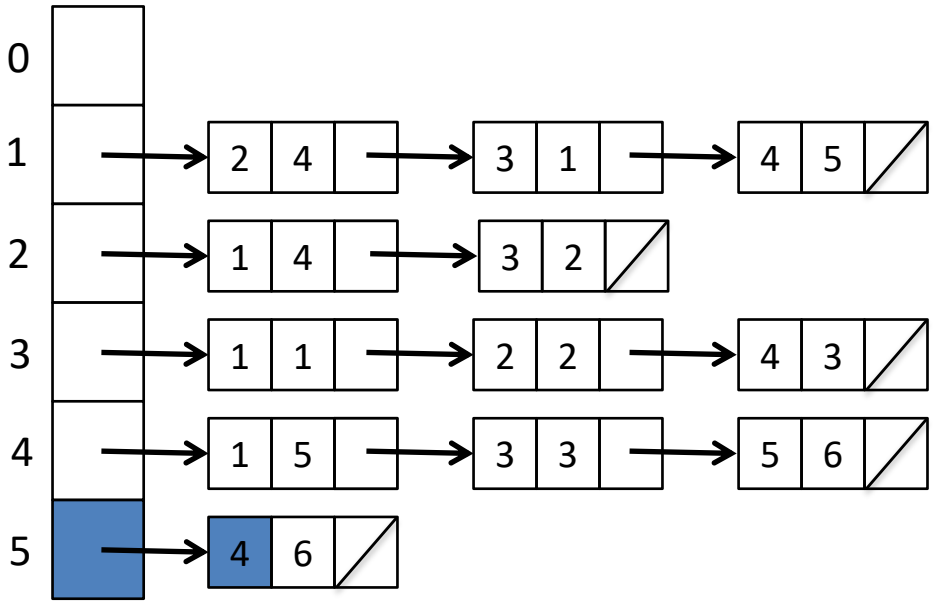
```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```



	intree	distance	parent
0			
1	T	0	-1
2	T	2	3
3	T	1	1
4	T	3	3
5	T	6	4

v	w	weight	dist
1	...	...	∞
1	4	6	4
2			1
3			∞
1			2
2			∞
1			3
4			∞
1			6
5			

```
for (i=1; i<=g->nvertices; i++) {
   intree[i] = FALSE;
   distance[i] = MAXINT;
   parent[i] = -1;
}

distance[start] = 0;
v = start;

while (intree[v] == FALSE) {

   intree[v] = TRUE;
   p = g->edges[v];

   while (p != NULL) {
      w = p->y;
      weight = p->weight;
      if ((weight < distance[w]) &&
          (intree[w] == FALSE)) {
         distance[w] = weight;
         parent[w] = v;
      }
      p = p->next;
   }

   v = 1;
   dist = MAXINT;
   for (i=1; i<=g->nvertices; i++)
      if ((intree[i] == FALSE) &&
          (distance[i] < dist)) {
         dist = distance[i];
         v = i;
      }
}
```

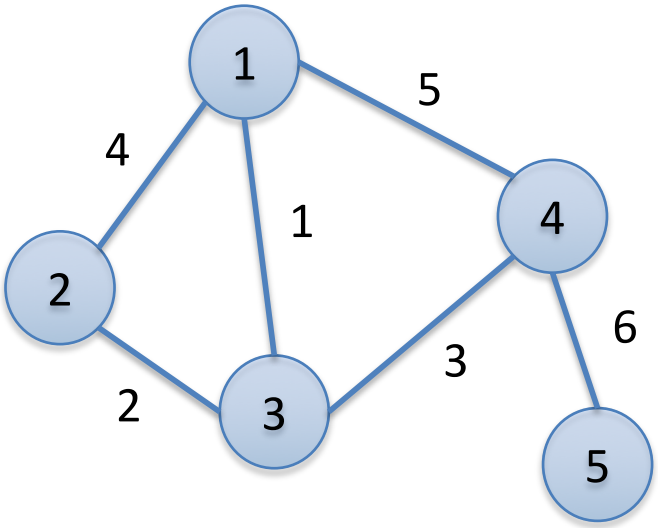


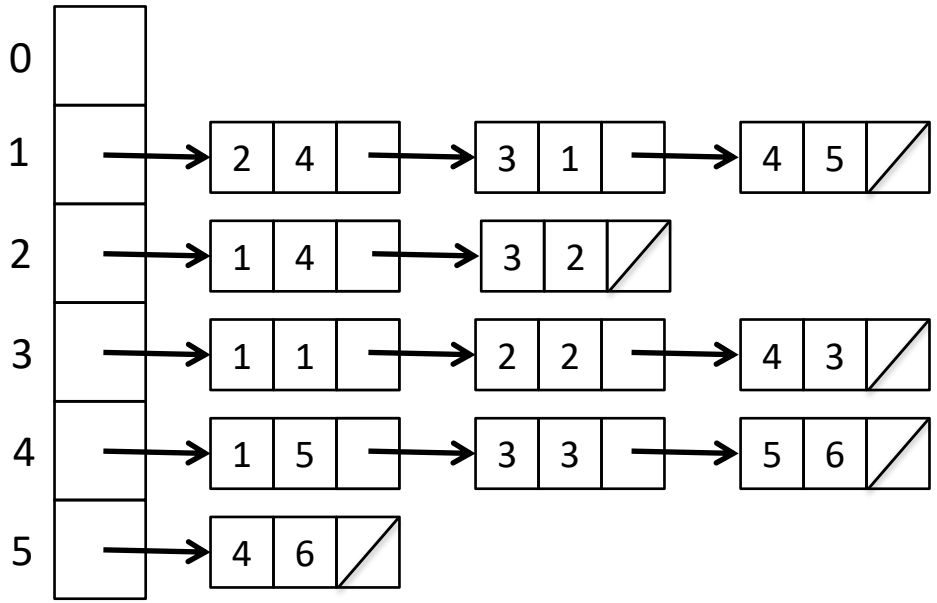
	intree	distance	parent
0			
1	T	0	-1
2	T	2	3
3	T	1	1
4	T	3	3
5	T	6	4

v	w	weight	dist
1	...	...	∞
1	4	6	4
2			1
3			∞
1			2
2			∞
1			3
4			∞
1			6
5			∞
1			

Minimum Spanning Tree

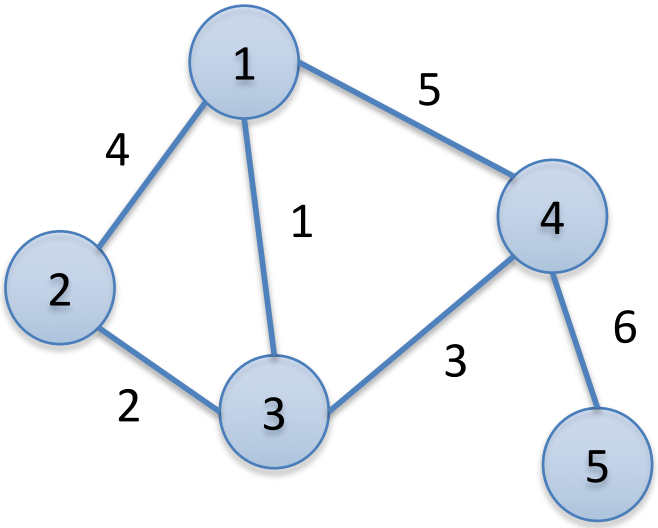
- Prim's Algorithm
  - The minimum spanning tree itself or its cost has to be reconstructed
    - Augment this procedure with statements that print the edges as they are found or totals the weight of all selected edges
    - Alternatively, the tree topology is given by the the `parent` relations and so it can be constructed using the original graph

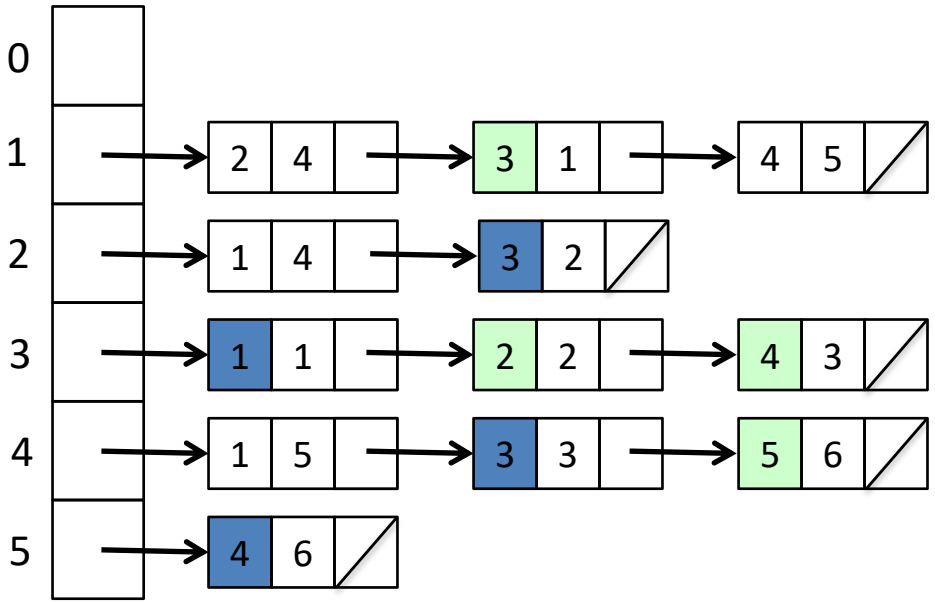




parent

0	
1	-1
2	3
3	1
4	3
5	4





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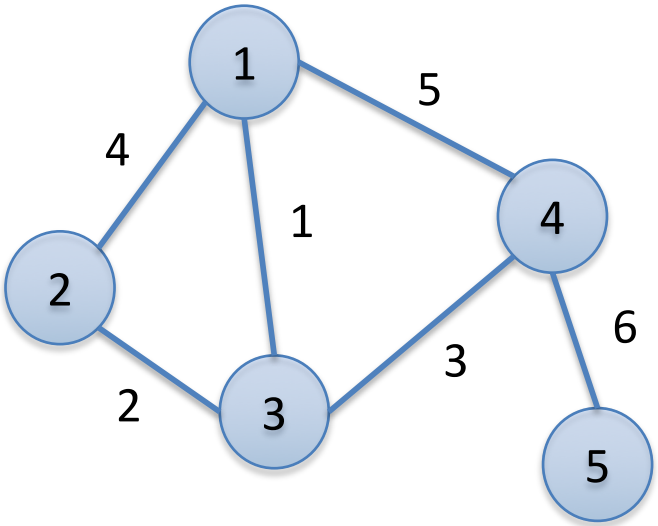
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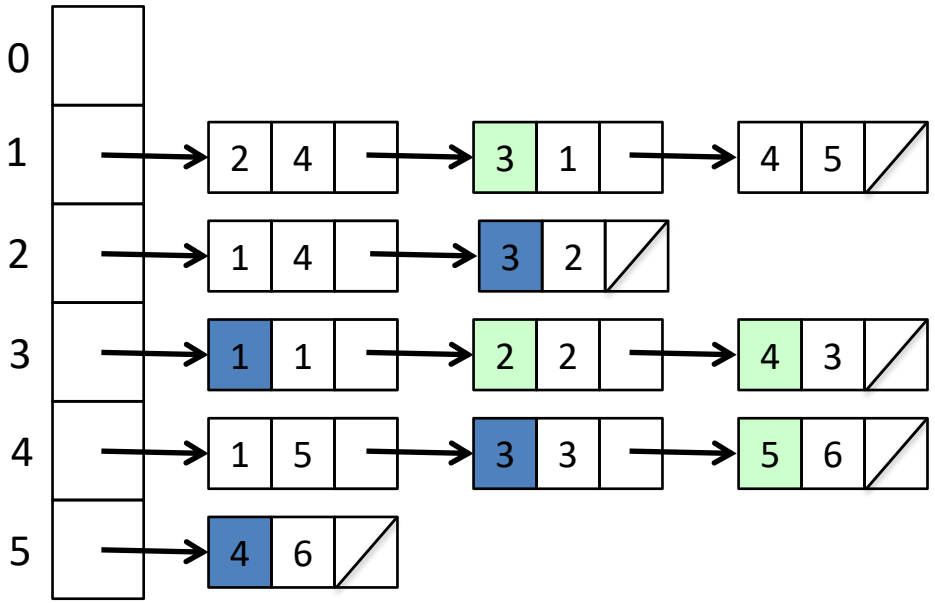
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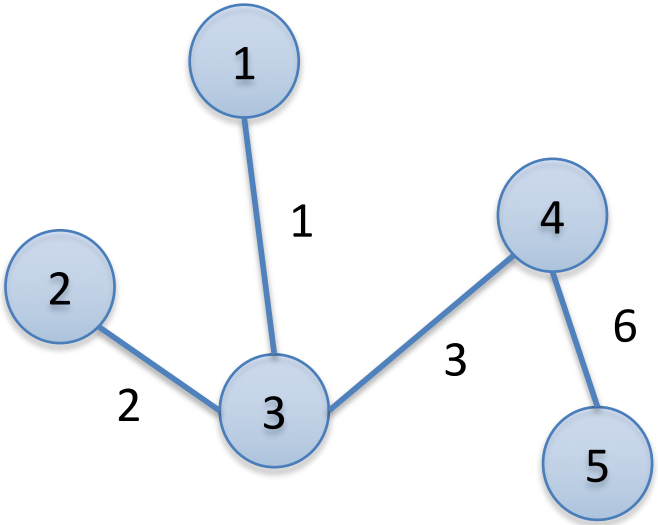
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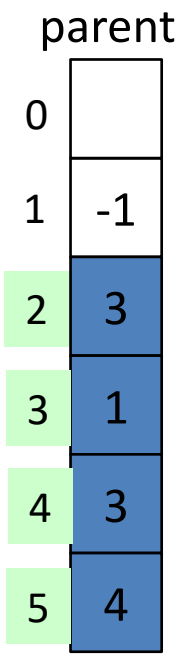
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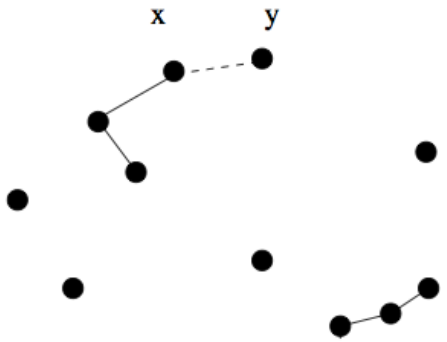
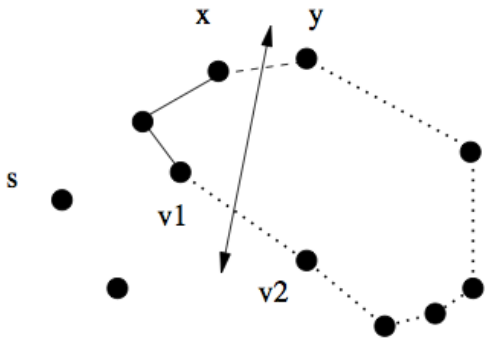
Minimum Spanning Tree

- Kruskal's Algorithm
  - Greedy algorithm:
    - Builds up connected components of vertices, culminating in the MST
    - Initially each vertex forms its own connected component
    - The algorithm repeatedly considers the lightest remaining edge
      - If its two endpoints lie within the same connected component, discard it (because adding it would create a cycle in the tree)
      - Otherwise, insert the edge and merge the two components in to one





Minimum Spanning Tree

- Kruskal's Algorithm

```
Kruskal-MST(G):  // note: no start vertex s; cf Prim's Algorithm

Put the edges in a priority queue ordered by weight
count = 0
While (count < n-1) do
        get next edge (v, w)
        if (component (v) != component (w))
                add to T_Kruskal
                merge component (v) and component (w)
```

Minimum Spanning Tree

- Prim's Algorithm vs. Kruskal's Algorithm
  - Prim $O(mn)$ … if we don't keep track of lightest edge
  - Prim $O(n^2)$ … if we do but use a simple data structure
  - Prim $O(m + n \log n)$ … if we use a priority queue data structure
  - Kruskal $O(m \log m)$

    n vertices, m edges

  - Prim's algorithm is faster on dense graphs (depends on n)
  - Kruskal's algorithm is faster on sparse graphs (depends on m)