Robotics: Principles and Practice

Module 3: Mobile Robots

Lecture 4: Relative position estimation using odometry

David Vernon
Carnegie Mellon University Africa

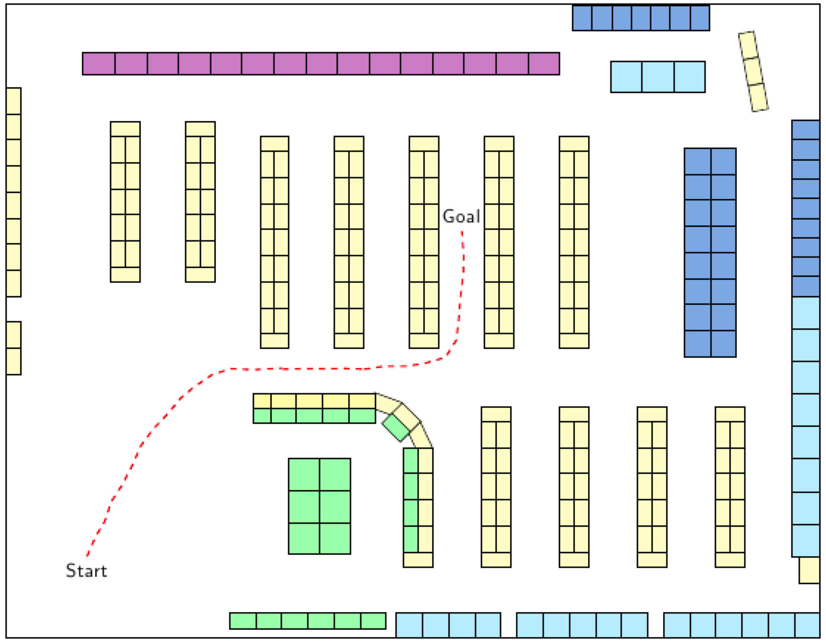
www.vernon.eu

Localization: The Position Estimation Problem

Two approaches

1. Absolute position estimation

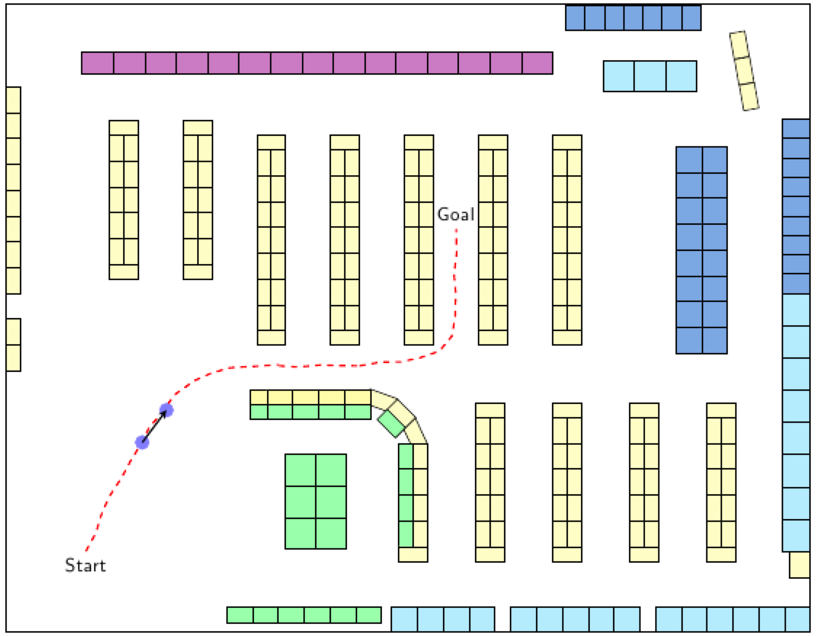
2. Relative position estimation



Relative Position Estimation

Detect the change in the position and orientation of the robot: $(\Delta x, \Delta y, \Delta \theta)$

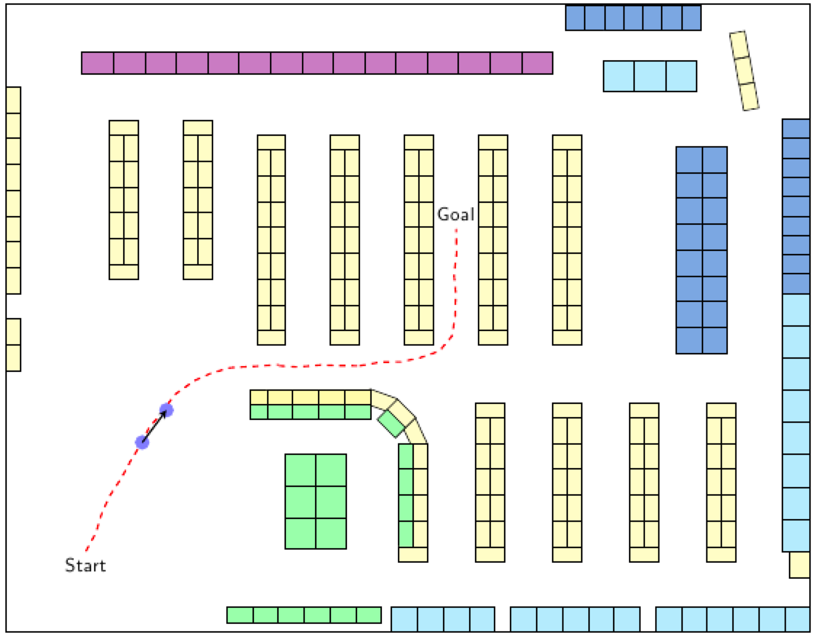
Combine the previous estimate and the change to determine the new estimate of the robot position



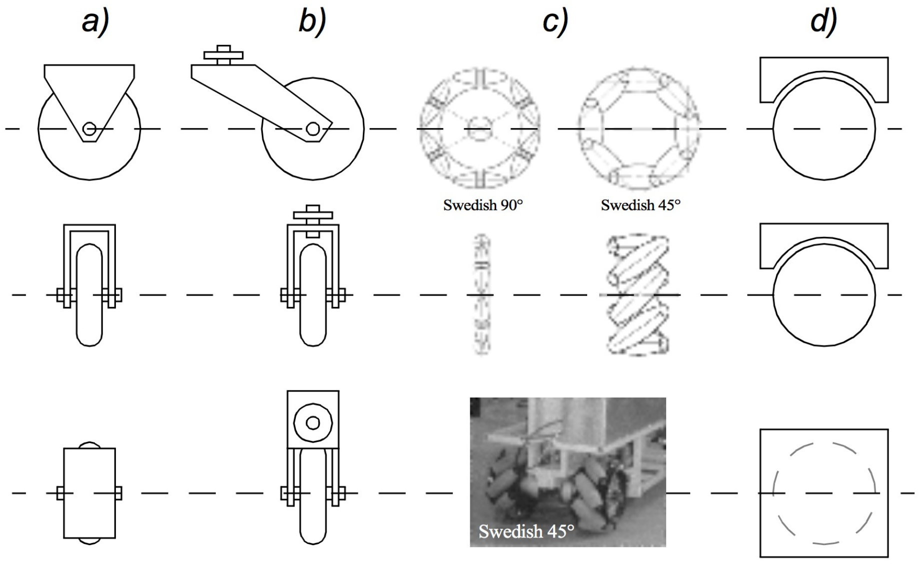
Relative Position Estimation

Options for detecting change in relative position:

- Inertial sensors
- Odometry



Wheeled Locomotion



We will focus on wheeled locomotion

Source: R. Siegwart and I. R. Nourbakhsh, *Introduction to Autonomous Mobile Robots*, MIT Press, 2004

(a) Standard wheel
(b) Castor wheel
(c) Swedish wheel
(d) Ball or spherical wheel

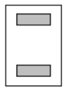
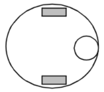
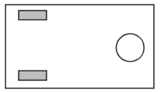
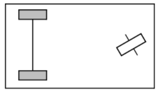
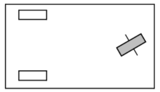
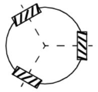
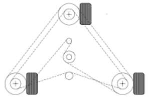
Rotation about axle for movement and about contact point for steering

Rotation about axle for movement and about vertical axis for steering; imparting a force on the robot body when steering

Rotation about axle for movement but also about rollers allowing movement is any direction

Omnidirectional wheel: can spin in any direction

Wheeled Locomotion

# of wheels	Arrangement	Description	Typical examples
2		One steering wheel in the front, one traction wheel in the rear	Bicycle, motorcycle
		Two-wheel differential drive with the center of mass (COM) below the axle	Cye personal robot
3		Two-wheel centered differential drive with a third point of contact	Nomad Scout, smartRob EPFL
		Two independently driven wheels in the rear/front, 1 unpowered omnidirectional wheel in the front/rear	Many indoor robots, including the EPFL robots Pygmalion and Alice
		Two connected traction wheels (differential) in rear, 1 steered free wheel in front	Piaggio minitrucks
		Two free wheels in rear, 1 steered traction wheel in front	Neptune (Carnegie Mellon University), Hero-1
		Three motorized Swedish or spherical wheels arranged in a triangle; omnidirectional movement is possible	Stanford wheel Tribolo EPFL, Palm Pilot Robot Kit (CMU)
		Three synchronously motorized and steered wheels; the orientation is not controllable	"Synchro drive" Denning MRV-2, Georgia Institute of Technology, I-Robot B24, Nomad 200

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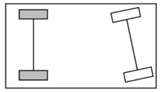
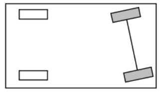
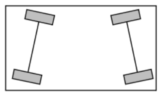
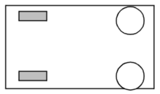
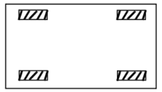
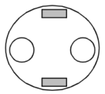
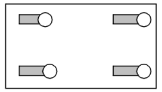
4		Two motorized wheels in the rear, 2 steered wheels in the front; steering has to be different for the 2 wheels to avoid slipping/skidding.	Car with rear-wheel drive
		Two motorized and steered wheels in the front, 2 free wheels in the rear; steering has to be different for the 2 wheels to avoid slipping/skidding.	Car with front-wheel drive
		Four steered and motorized wheels	Four-wheel drive, four-wheel steering Hyperion (CMU)
		Two traction wheels (differential) in rear/front, 2 omnidirectional wheels in the front/rear	Charlie (DMT-EPFL)
		Four omnidirectional wheels	Carnegie Mellon Uranus
		Two-wheel differential drive with 2 additional points of contact	EPFL Khepera, Hyperbot Chip
		Four motorized and steered castor wheels	Nomad XR4000

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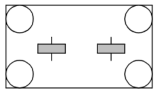
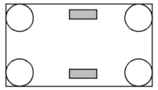
6		Two motorized and steered wheels aligned in center, 1 omnidirectional wheel at each corner	First
		Two traction wheels (differential) in center, 1 omnidirectional wheel at each corner	Terregator (Carnegie Mellon University)

Icons for the each wheel type are as follows:	
	unpowered omnidirectional wheel (spherical, castor, Swedish);
	motorized Swedish wheel (Stanford wheel);
	unpowered standard wheel;
	motorized standard wheel;
	motorized and steered castor wheel;
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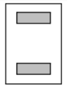
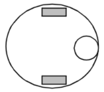
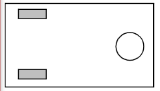
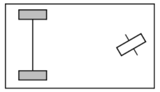
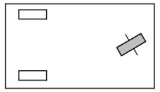
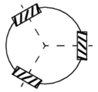
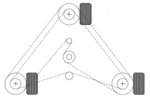
		Two-wheel differential drive with the center of mass (COM) below the axle	Cye personal robot
3		Two-wheel centered differential drive with a third point of contact	Nomad Scout, smartRob EPFL
		Two independently driven wheels in the rear/front, 1 unpowered omnidirectional wheel in the front/rear	Many indoor robots, including the EPFL robots Pygmalion and Alice
		Two connected traction wheels (differential) in rear, 1 steered free wheel in front	Piaggio minitrucks
		Two free wheels in rear, 1 steered traction wheel in front	Neptune (Carnegie Mellon University), Hero-1
		Three motorized Swedish or spherical wheels arranged in a triangle; omnidirectional movement is possible	Stanford wheel Tribolo EPFL, Palm Pilot Robot Kit (CMU)
		Three synchronously motorized and steered wheels; the orientation is not controllable	"Synchro drive" Denning MRV-2, Georgia Institute of Technology, I-Robot B24, Nomad 200

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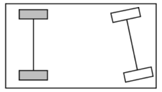
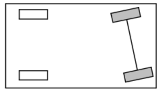
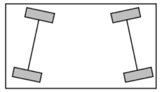
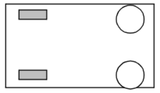
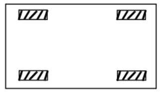
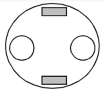
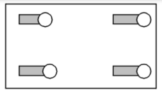
4		Two motorized wheels in the rear, 2 steered wheels in the front; steering has to be different for the 2 wheels to avoid slipping/skidding.	Car with rear-wheel drive
		Two motorized and steered wheels in the front, 2 free wheels in the rear; steering has to be different for the 2 wheels to avoid slipping/skidding.	Car with front-wheel drive
		Four steered and motorized wheels	Four-wheel drive, four-wheel steering Hyperion (CMU)
		Two traction wheels (differential) in rear/front, 2 omnidirectional wheels in the front/rear	Charlie (DMT-EPFL)
		Four omnidirectional wheels	Carnegie Mellon Uranus
		Two-wheel differential drive with 2 additional points of contact	EPFL Khepera, Hyperbot Chip
		Four motorized and steered castor wheels	Nomad XR4000

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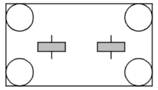
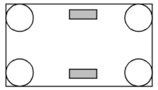
6		Two motorized and steered wheels aligned in center, 1 omnidirectional wheel at each corner	First
		Two traction wheels (differential) in center, 1 omnidirectional wheel at each corner	Terregator (Carnegie Mellon University)

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We will study two-wheel differential drive locomotion

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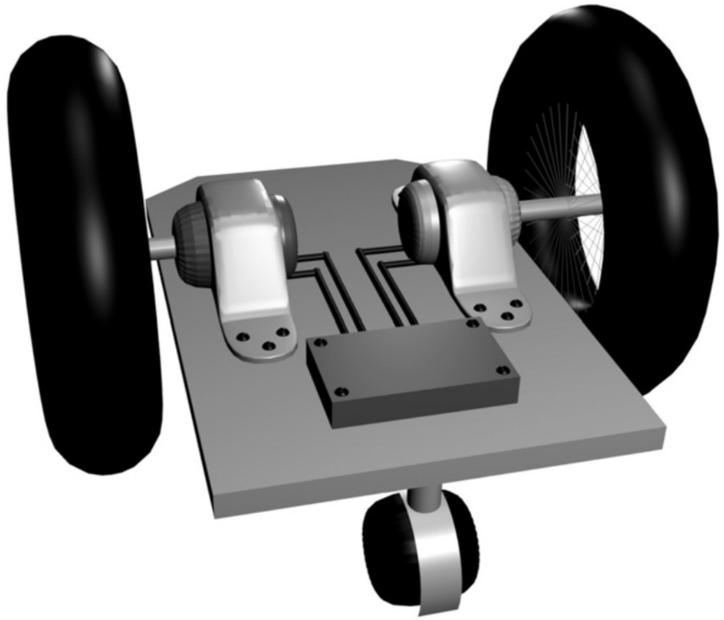
Wheeled Locomotion



Source: M. Mataric, The Robotics Primer, MIT Press, 2007

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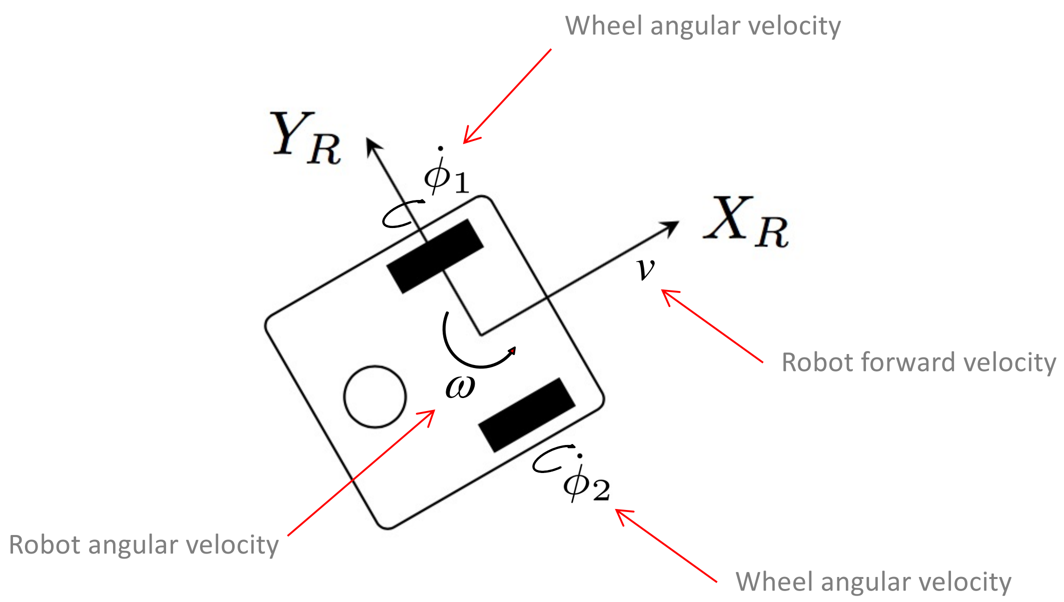


Odometry-based Position Estimation

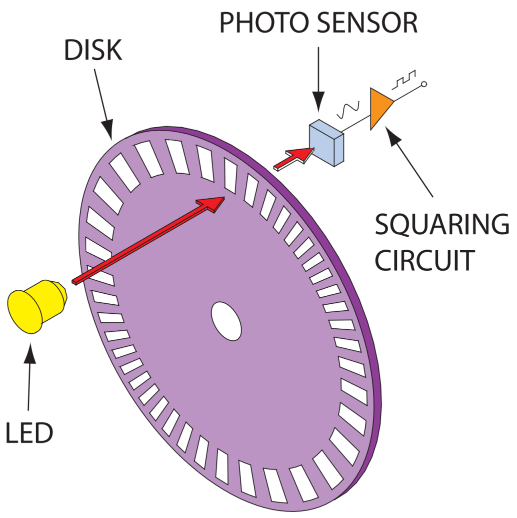
Overall approach

- Measure the angular velocities of the left and right wheels $\dot{\phi}_1 \quad \dot{\phi}_2$
- Compute the instantaneous velocities in the robot frame of reference
- Compute the displacement and change in orientation (in a given time interval) in the robot frame of reference R
- Compute the displacement and change in orientation (in a given time interval) in the global frame of reference (inertial frame of reference I)
- Update the position of the robot with respect to its previous position
- Repeat update (e.g. every 100 ms)

Angular velocities of the left and right wheels

Need to use proprioceptive sensors

- Typically, encoders on the wheels
- Go to http://encoder.com/videos/
  to watch a video explaining the operation of encoders



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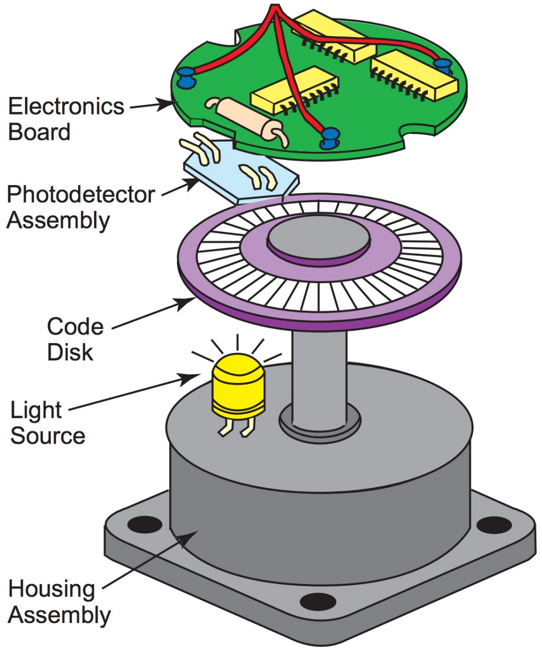

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Odometry-based Position Estimation

Overall approach

- Measure the angular velocities of the left and right wheels
- Compute the instantaneous velocities in the robot frame of reference — v, ω
- Compute the displacement and change in orientation (in a given time interval) in the robot frame of reference R — $\Delta\xi_R = (\Delta x, \Delta y, \Delta\theta)$
- Compute the displacement and change in orientation (in a given time interval) in the global frame of reference (inertial frame of reference I)
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Odometry-based Position Estimation

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$$\Delta\xi_R = (\Delta x, \Delta y, \Delta\theta)$$

$$\downarrow$$

$$\Delta\xi_I = (\Delta x_I, \Delta y_I, \Delta\theta)$$

Odometry-based Position Estimation

Overall approach

- Measure the angular velocities of the left and right wheels
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- Repeat update (e.g. every 100 ms)

$$\begin{aligned} x_I(t+1) &= x_I(t) + \Delta x_I \\ y_I(t+1) &= y_I(t) + \Delta y_I \\ \theta(t+1) &= \theta(t) + \Delta\theta \end{aligned}$$

Update the position of the robot with respect to its previous position

Assuming we know the position at time $t = 0$,
we estimate the position at time in the next time period $t = 1$ as follows

$$\begin{aligned} x_I(1) &= x_I(0) + \Delta x_I \\ y_I(1) &= y_I(0) + \Delta y_I \\ \theta(1) &= \theta(0) + \Delta\theta \end{aligned}$$

Odometry-based Position Estimation

Overall approach

- Measure the angular velocities of the left and right wheels
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- Compute the displacement and change in orientation (in a given time interval) in the robot frame of reference
- Compute the displacement and change in orientation in the global frame of reference (inertial frame of reference)
- Update the position of the robot with respect to its previous position
- Repeat update (e.g. every 100 ms)

Repeat Update

We update the position at each time period

$$
\begin{aligned}
x_I(t+1) &= x_I(t) + \Delta x_I \\
y_I(t+1) &= y_I(t) + \Delta y_I \\
\theta(t+1) &= \theta(t) + \Delta\theta
\end{aligned}
$$

Odometry-based Position Estimation

Overall approach

- Measure the angular velocities of the left and right wheels $\dot{\phi}_1 \quad \dot{\phi}_2$
- Compute the instantaneous velocities in the robot frame of reference $v,\ \omega$
- Compute the displacement and change in orientation (in a given time interval) in the robot frame of reference R $\quad \Delta\xi_R = (\Delta x, \Delta y, \Delta\theta)$
- Compute the displacement and change in orientation (in a given time interval) in the global frame of reference (inertial frame of reference I) $\quad \Delta\xi_I = (\Delta x_I, \Delta y_I, \Delta\theta)$
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\theta(t+1) &= \theta(t) + \Delta\theta
\end{aligned}
$$

Odometry-based Position Estimation

Limitations of Odometry – Sources of Error

- Assumptions
  - Assumes control variables (angular velocities) are constant
- Robot's geometry (systematic errors)
  - wheel imbalance: one wheel larger than the other
  - axis imbalance: one axis is longer than the other
  - actual wheel distance not the same as nominal one (D)
- Interaction with environment (non-systematic errors)
  - slippage of one wheel
  - bumps and gaps in the floor
- The main problem
  - Odometry error can grow without bounds

Odometry-based Position Estimation

Limitations – Sources of Error

- Difference in wheel radius E_r
  - the robot makes a curved instead of a straight path

    (because, for identical left and right angular velocities, the associated displacements will be different)

- Error in wheel distance E_l
  - the robot over-estimates or under-estimates the amount of turning

    (because the angle is a function of l, the wheel distance)

Summary

- Absolute position estimation (localization)
  - match observed features with prior knowledge (map)
  - requires good perception, good map
- Relative position estimation (position tracking)
  - update previous position by measuring displacement
  - inevitably drifts with time
- General solution: combined
  - absolute localization whenever possible
  - continuous position tracking in between localizations

Odometry-based Position Estimation

Now we need to know how to compute these three transformations:

- Measure the angular velocities of the left and right wheels — $\dot{\phi}_1 \quad \dot{\phi}_2$
- Compute the instantaneous velocities in the robot frame of reference — $v,\ \omega$
- Compute the displacement and change in orientation (in a given time interval) in the robot frame of reference R — $\Delta\xi_R = (\Delta x, \Delta y, \Delta\theta)$
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