Robotics: Principles and Practice

Module 5: Robot Vision

Lecture 8: Perspective transformation; camera model; camera calibration

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Homogeneous Coordinates

A 3D vector, $v = ai + bj + ck$, where i, j and k are unit vectors along the X, Y and Z axes is represented in homogenous co-ordinates as

$$v = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

where $a = \frac{x}{w}, b = \frac{y}{w}$ and $c = \frac{z}{w}$

Homogeneous Coordinates

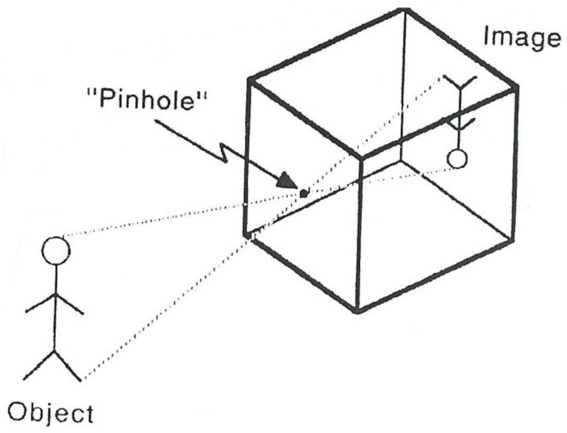
Thus, the additional fourth co-ordinate w is just a scaling factor and means that a single 3D vector can be represented by several homogenous co-ordinates

For example, $3i + 4j + 5k$ can be represented by $\begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \end{bmatrix}$ or by $\begin{bmatrix} 6 \\ 8 \\ 10 \\ 2 \end{bmatrix}$.

Note that, since division of zero is indeterminate, the vector $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ is undefined.

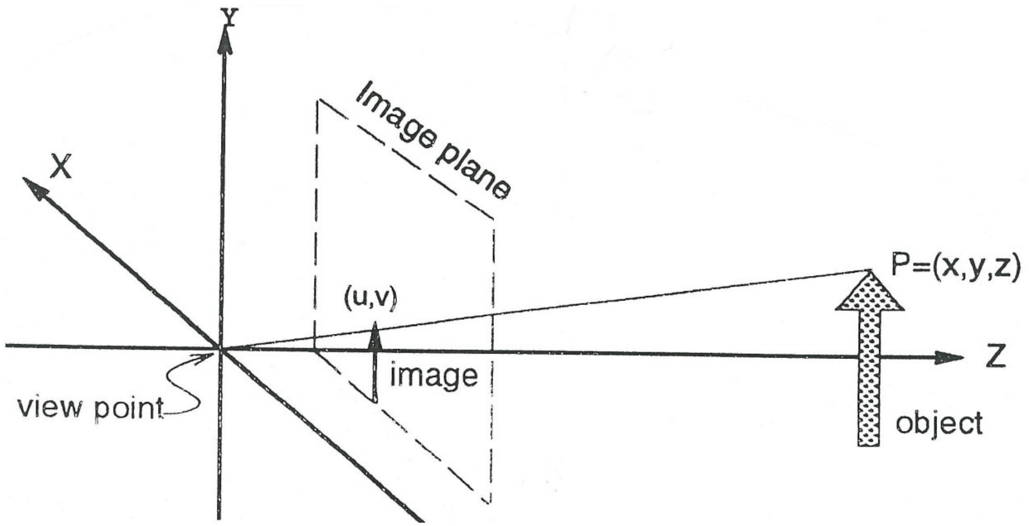
Perspective Projection

Pinhole model of a camera



Perspective Projection

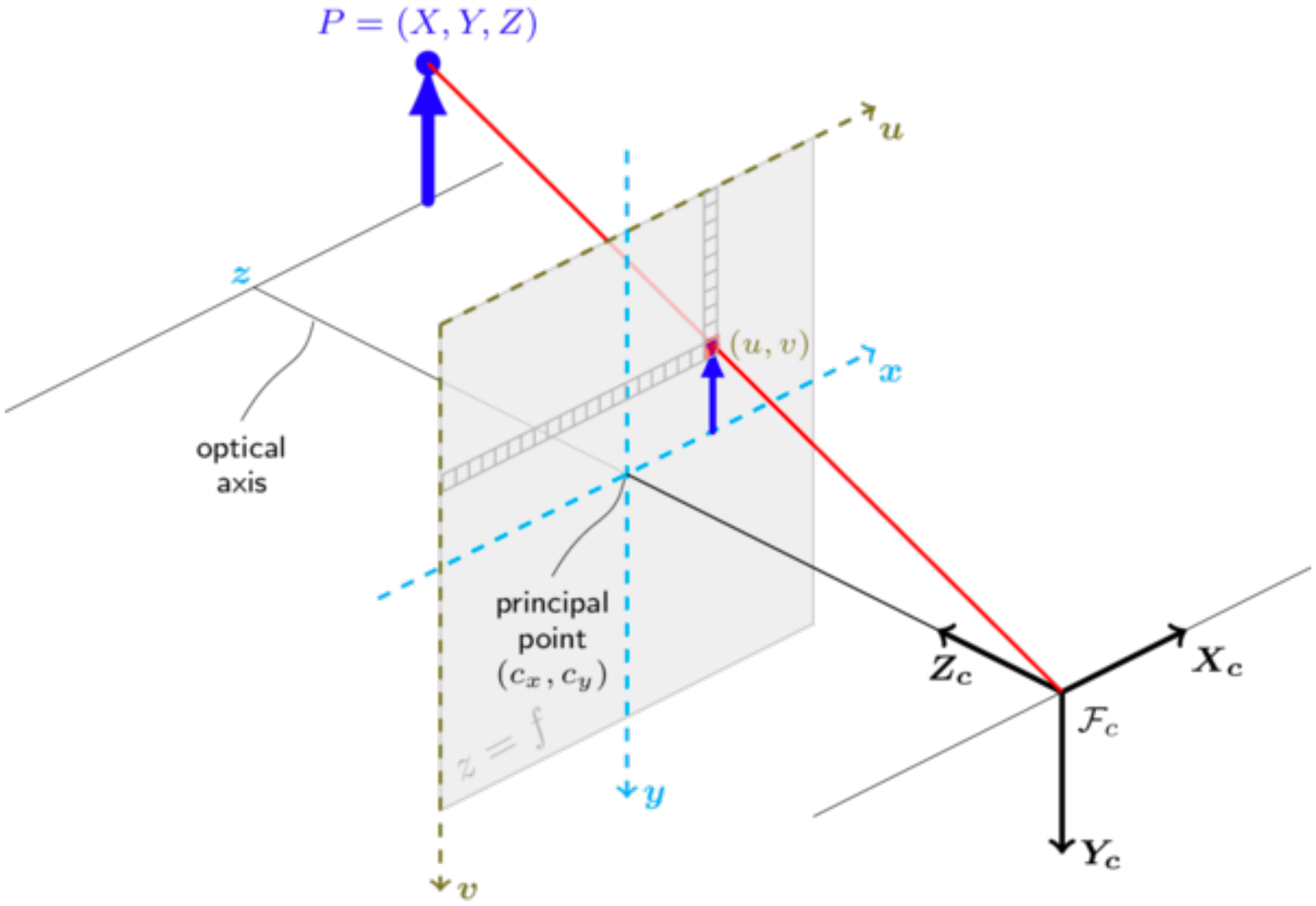
Pinhole model of a camera



$$u = \frac{f}{z} x \qquad v = \frac{f}{z} y$$

Source: Markus Vincze, Technische Universität Wien

Perspective Projection



http://docs.opencv.org/2.4/modules/calib3d/doc/camera_calibration_and_3d_reconstruction.html

Perspective Projection

For any given optical configuration, there are two aspects to the relationship between world point and image point

- the camera model, which maps a 3D world point to its corresponding 2D image point

- the inverse perspective transformation, which is used to identify the 3D world point(s) corresponding to a particular 2D image point.

Perspective Projection

Since the imaging process is a projection (from a 3D world to a 2D image), the inverse process, *i.e.* the inverse perspective transformation, cannot uniquely determine a single world point for a given image point

- the inverse perspective transformation thus maps a 2D image point into a line (an infinite set of points) in the 3D world,
- however, it does so in a useful and well-constrained manner

Perspective Projection

For the following, we will assume that the camera model (and, hence, the inverse perspective transformation) is linear

This treatment closely follows that of [Ballard and Brown 1982]

The Camera Model

Let the image point in question be given by the co-ordinates $\begin{bmatrix} U \\ V \end{bmatrix}$ which, in homogenous co-ordinates is written $\begin{bmatrix} u \\ v \\ t \end{bmatrix}$.

Thus, $U = \dfrac{u}{t}$

and $V = \dfrac{v}{t}$

The Camera Model

Let the desired camera model, a transformation which maps the 3D world point to the corresponding 2D image point, be C.

Thus,

$$C \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ t \end{bmatrix}$$

The Camera Model

Hence C must be a 3*4 (homogenous) transformation

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \end{bmatrix}$$

and

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ t \end{bmatrix}$$

The Camera Model

Expanding this matrix equation, we get :

$$C_{11}x + C_{12}y + C_{13}z + C_{14} = u \quad (1)$$

$$C_{21}x + C_{22}y + C_{23}z + C_{24} = v \quad (2)$$

$$C_{31}x + C_{32}y + C_{33}z + C_{34} = t \quad (3)$$

The Camera Model

but $u = Ut$ and $v = Vt$

so
$$\begin{aligned} u - Ut &= 0 \quad (4) \\ v - Vt &= 0 \quad (5) \end{aligned}$$

Substituting (1) and (3) for u and t, respectively, in (4) and substituting (2) and (3) for v and t, respectively, in (5) :

$$\begin{aligned} C_{11}x + C_{12}y + C_{13}z + C_{14}x - UC_{31}x - UC_{32}y - UC_{33}z - UC_{34} &= 0 \quad (6) \\ C_{21}x + C_{22}y + C_{23}z + C_{24}x - VC_{31}x - VC_{32}y - VC_{33}z - VC_{34} &= 0 \quad (7) \end{aligned}$$

The Camera Model

If we establish this association

i.e. if we measure the values of x, y, z, U and V

we will have two equations in which the only unknowns are the twelve camera model coefficients (and which we want to identify)

Since a single observation gives rise to two equations, six observations will produce twelve simultaneous equations which we can solve for the required camera coefficients C_{ij}.

The Camera Model

Remember that these two equations arose from the association of a particular world point

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

and a corresponding image point $\begin{bmatrix} U \\ V \end{bmatrix}$

The Camera Model

Before we proceed, however, we need to note that the overall scaling of C is irrelevant due to the homogenous formulation and, thus, the value of C_{34} may be set arbitrarily to 1 and we can re-write (6) and (7), completing the equations so that terms for each coefficient of C is included, as follows

$$\begin{aligned}
C_{11}x + C_{12}y + C_{13}z + C_{14}x + C_{21}0 + C_{22}0 + C_{23}0 + C_{24}0 - UC_{31}x - UC_{32}y - UC_{33}z &= U \\
C_{11}0 + C_{12}0 + C_{13}0 + C_{14}0 + C_{21}x + C_{22}y + C_{23}z + C_{24}x - VC_{31}x - VC_{32}y - VC_{33}z &= V
\end{aligned}$$

This reduces the number of unknowns to eleven

The Camera Model

For six observations, we now have twelve equations and eleven unknowns:
i.e. the system of equations is over-determined

Re-formulating the twelve equations in matrix form, we can obtain
a least-square-error solution to the system using the pseudo-inverse method

The Camera Model

Let

$$X = \begin{bmatrix} x^1 & y^1 & z^1 & 1 & 0 & 0 & 0 & 0 & -U^1x^1 & -U^1y^1 & -U^1z^1 \\ 0 & 0 & 0 & 0 & x^1 & y^1 & z^1 & 1 & -V^1x^1 & -V^1y^1 & -V^1z^1 \\ x^2 & y^2 & z^2 & 1 & 0 & 0 & 0 & 0 & -U^2x^2 & -U^2y^2 & -U^2z^2 \\ 0 & 0 & 0 & 0 & x^2 & y^2 & z^2 & 1 & -V^2x^2 & -V^2y^2 & -V^2z^2 \\ x^3 & y^3 & z^3 & 1 & 0 & 0 & 0 & 0 & -U^3x^3 & -U^3y^3 & -U^3z^3 \\ 0 & 0 & 0 & 0 & x^3 & y^3 & z^3 & 1 & -V^3x^3 & -V^3y^3 & -V^3z^3 \\ x^4 & y^4 & z^4 & 1 & 0 & 0 & 0 & 0 & -U^4x^4 & -U^4y^4 & -U^4z^4 \\ 0 & 0 & 0 & 0 & x^4 & y^4 & z^4 & 1 & -V^4x^4 & -V^4y^4 & -V^4z^4 \\ x^5 & y^5 & z^5 & 1 & 0 & 0 & 0 & 0 & -U^5x^5 & -U^5y^5 & -U^5z^5 \\ 0 & 0 & 0 & 0 & x^5 & y^5 & z^5 & 1 & -V^5x^5 & -V^5y^5 & -V^5z^5 \\ x^6 & y^6 & z^6 & 1 & 0 & 0 & 0 & 0 & -U^6x^6 & -U^6y^6 & -U^6z^6 \\ 0 & 0 & 0 & 0 & x^6 & y^6 & z^6 & 1 & -V^6x^6 & -V^6y^6 & -V^6z^6 \end{bmatrix}$$

The Camera Model

Let

$$c = \begin{bmatrix} C_{11} \\ C_{12} \\ C_{13} \\ C_{14} \\ C_{21} \\ C_{22} \\ C_{23} \\ C_{24} \\ C_{31} \\ C_{32} \\ C_{33} \end{bmatrix} \qquad y = \begin{bmatrix} U^1 \\ V^1 \\ U^2 \\ V^2 \\ U^3 \\ V^3 \\ U^4 \\ V^4 \\ U^5 \\ V^5 \\ U^6 \\ V^6 \end{bmatrix}$$

thus,

$$Xc = y$$

The trailing superscript denotes the observation number

The Camera Model

Then :

$$c = \left(X^{\top} X\right)^{-1} y$$

$$c = X^{\dagger} y$$

We assumed above that we make six observations to establish the relationship between six sets of image co-ordinates and six sets of real world co-ordinates.

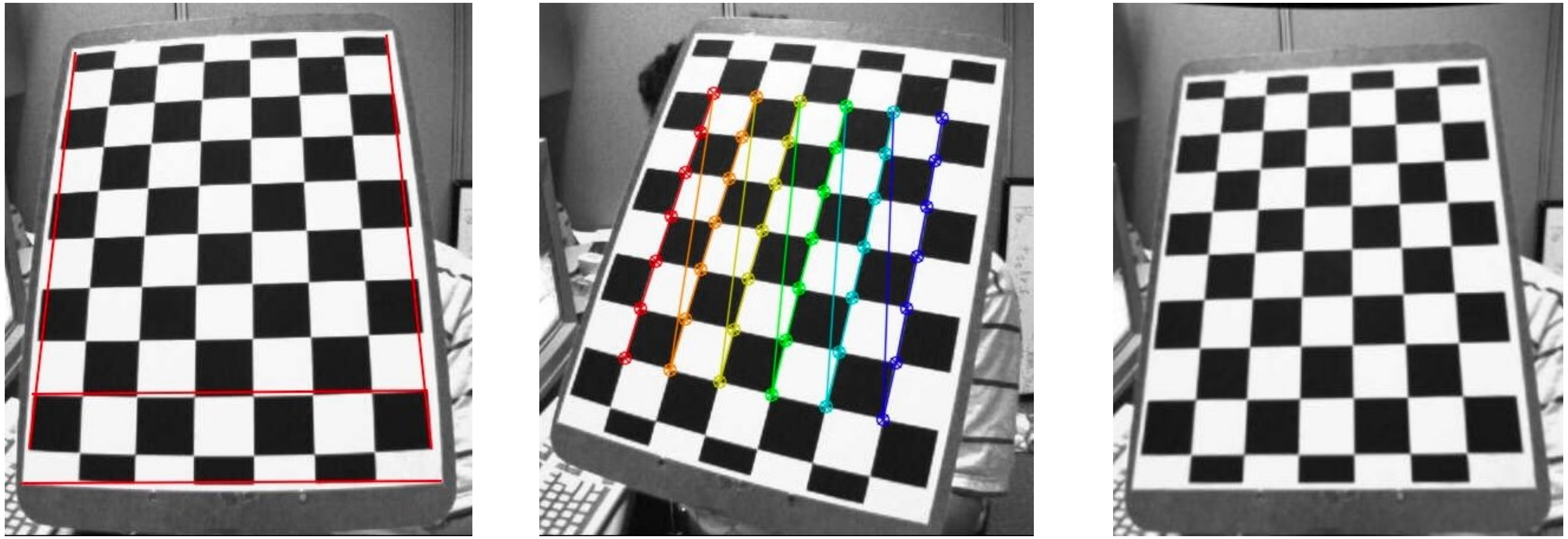
In general, it is better to significantly over-determine the system of equations by generating a larger set of observations than the minimal six

The Camera Model

- This is, in fact, the central issue in the derivation of the camera model, that is, the identification of a set of corresponding control points

- There are several approaches.
  - we could present the imaging system with a calibration grid
  - empirically measure the positions of the grid intersections
  - identifying the corresponding points in the image, either interactively or automatically

The Camera Model



http://docs.opencv.org/3.1.0/dc/dbb/tutorial_py_calibration.html

The Camera Model

- When used for robot manipulation, the empirical measurement of these real-world co-ordinates may be prone to error and this error will be manifested in the resultant camera model

- It is better practice to get the robot itself to calibrate the system

  – by fitting it with an end-effector with an accurately located calibration mark (*e.g.* a cross-hair or a surveyor's mark)

  – by programming it to place the mark at a variety of (known) positions in the field of view of the camera system

The Camera Model

Main benefit:

The two components of the manipulation environment,

the robot and the vision system

both of which are reasoning about co-ordinates in the 3D world, are effectively coupled

So, if the vision system "sees" something at a particular location, that is where the robot manipulator will go

The Camera Model

So far:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ t \end{bmatrix}$$

The Camera Model

Intrinsic and extrinsic camera parameters

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ t \end{bmatrix}$$

(2D point (3x1)) = (Camera to pixel coordinate transformation matrix (3x3)) (Perspective projection matrix (3x4)) (World to camera coordinate transformation matrix (4x4)) (3D point (4x1))

Credit: Markus Vincze, Technische Universität Wien

The Camera Model

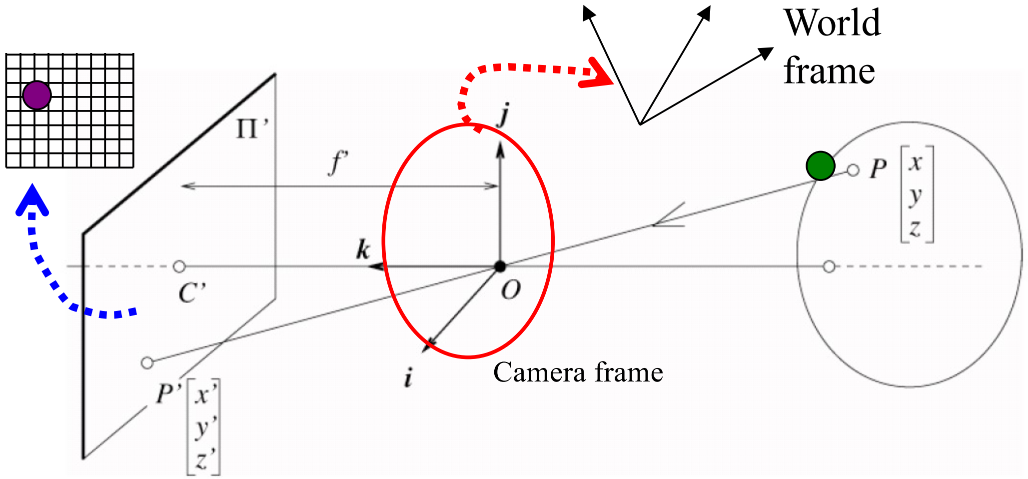
Extrinsic camera parameters

Transformation from world frame of reference to camera frame of reference

(2D point (3x1)) = (Camera to pixel coordinate transformation matrix (3x3)) (Perspective projection matrix (3x4)) (World to camera coordinate transformation matrix (4x4)) (3D point (4x1))

Credit: Markus Vincze, Technische Universität Wien

The Camera Model



Credit: Markus Vincze, Technische Universität Wien

The Camera Model

Intrinsic camera parameters

- The perspective projection (only parameter is the focal length f)
- The transformation between camera frame coordinates and pixel coordinates
- The geometric distortion introduced by the optics

$$\begin{pmatrix} \text{2D point} \\ (3\times1) \end{pmatrix} = \underbrace{\begin{pmatrix} \text{Camera to pixel coordinate transformation matrix} \\ (3\times3) \end{pmatrix} \begin{pmatrix} \text{Perspective projection matrix} \\ (3\times4) \end{pmatrix}}_{\text{intrinsic}} \begin{pmatrix} \text{World to camera coordinate transformation matrix} \\ (4\times4) \end{pmatrix} \begin{pmatrix} \text{3D point} \\ (4\times1) \end{pmatrix}$$

Credit: Markus Vincze, Technische Universität Wien

The Camera Model

Perspective Projection

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

(2D point (3x1)) = (Camera to pixel coordinate transformation matrix (3x3)) (Perspective projection matrix (3x4)) (World to camera coordinate transformation matrix (4x4)) (3D point (4x1))

Credit: Markus Vincze, Technische Universität Wien

The Camera Model

Transformation from camera to pixel coordinates

$$(x, y) \longrightarrow (x_{im}, y_{im})$$

$$x = -\,(x_{im} - c_x)\, s_x$$

$$y = -\,(y_{im} - c_y)\, s_y$$

(c_x, c_y)
coordinates of the image centre / principal point
(intersection of principal ray with image)

(s_x, s_y)
effective size of the pixel, in millimetres,
in the horizontal and vertical directions

(2D point (3x1)) = (Camera to pixel coordinate transformation matrix (3x3)) (Perspective projection matrix (3x4)) (World to camera coordinate transformation matrix (4x4)) (3D point (4x1))

Credit: Trucco and Verri 1998

The Camera Model

Correction of radial distortion

$$x = x_d (1 + k_1 r^2 + k_2 r^4)$$

$$y = y_d (1 + k_1 r^2 + k_2 r^4)$$

$$r^2 = x_d^2 + y_d^2$$

(x_d, y_d)
coordinates of the distorted point

k_1 and k_2 are further intrinsic parameters

The magnitude of the geometric distortion depends on the quality of the lens

(2D point (3x1)) = (Camera to pixel coordinate transformation matrix (3x3)) (Perspective projection matrix (3x4)) (World to camera coordinate transformation matrix (4x4)) (3D point (4x1))

Credit: Trucco and Verri 1998

The Camera Model

$k_1 > 0$ $\qquad$ $k_1 < 0$

Credit: Markus Vincze, Technische Universität Wien

Reading

R. Szeliski, *Computer Vision: Algorithms and Applications*, Springer, 2010.

Section 2.1.5 3D to 2D projections

Section 6.3 Geometric intrinsic calibration

Vernon, D. 1991. Machine Vision: Automated Visual Inspection and Robot Vision, Prentice-Hall International; Section 8.6

OpenCV documentation on camera calibration:

http://docs.opencv.org/2.4/modules/calib3d/doc/camera_calibration_and_3d_reconstruction.html

Demo

Read "Camera Modelling and Camera Calibration.pdf"
Then walk through the following example applications:

cameraCalibration

cameraModelData

cameraModel