Introduction to Cognitive Robotics

Module 4: Robot Manipulators

Lecture 2: Object pose specification with homogenous transformations and vectors & quaternions

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Recall, we have developed a system where we can

specify the position and orientation of coordinate reference frames anywhere

w.r.t. station frame of reference

with respect to each other

or

with respect to a given base frame

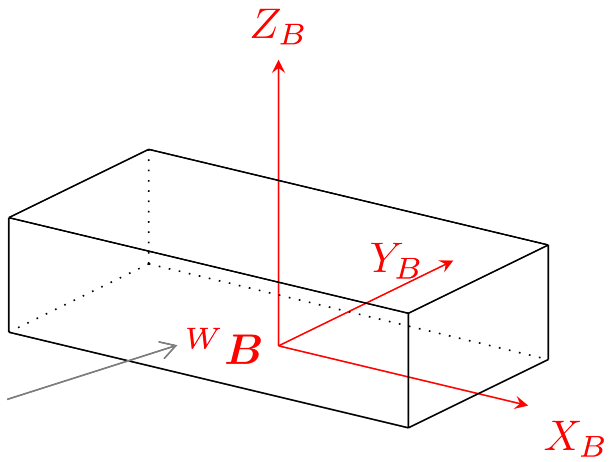
w.r.t. fixed world frame of reference

This, in itself, is not much use since the world you and I know does not have too many coordinate reference frames in it

What we really require is a way of identifying the pose of objects

Position and Orientation:
Six degrees of freedom

The trick, and it is no more than a trick, is to attach a coordinate frame to an object, *i.e.* symbolically glue an XYZ frame into an object simply by defining it to be there

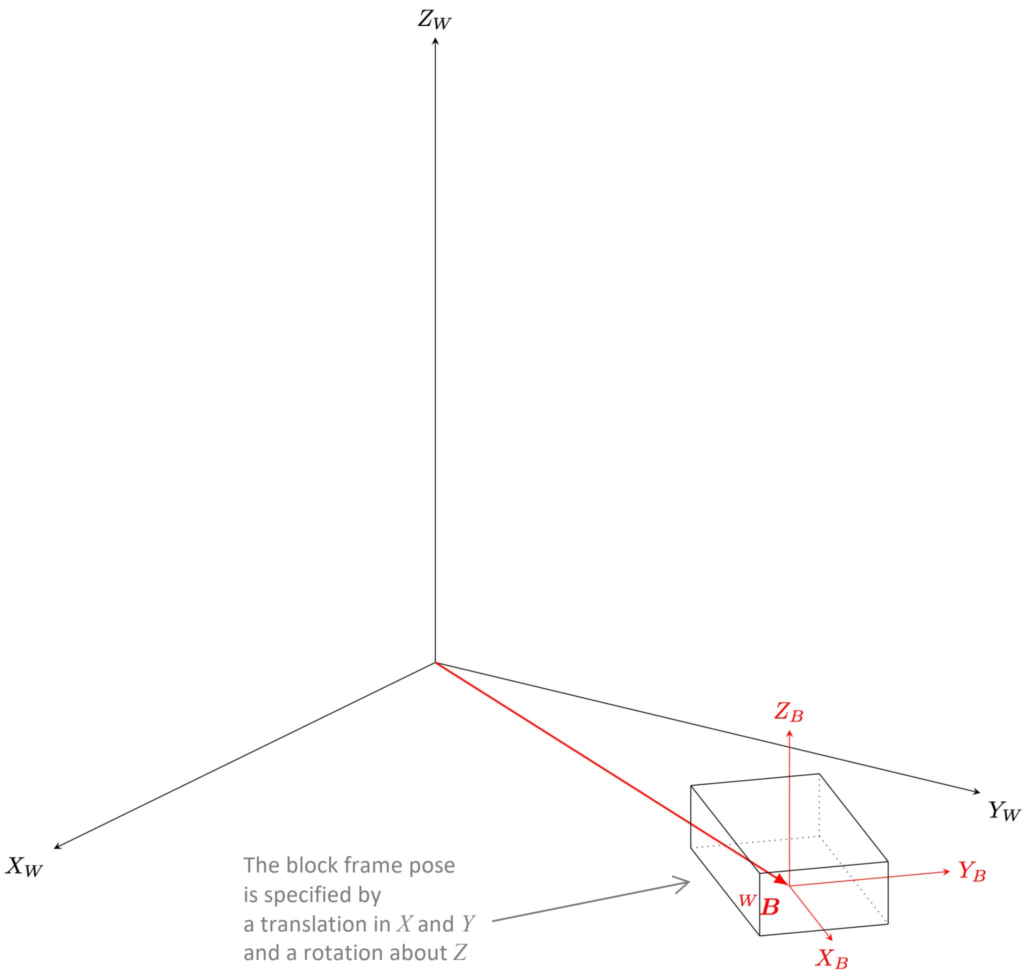


The block frame is defined with respect to the world frame of reference

- As we rotate and translate the coordinate frame, so we rotate and translate objects

- We can arbitrarily position and orient a coordinate frame – and an object – by specifying the required translations and rotations

- Thus, we specify the pose of an object by specifying its associated coordinate frame (homogeneous transformation)



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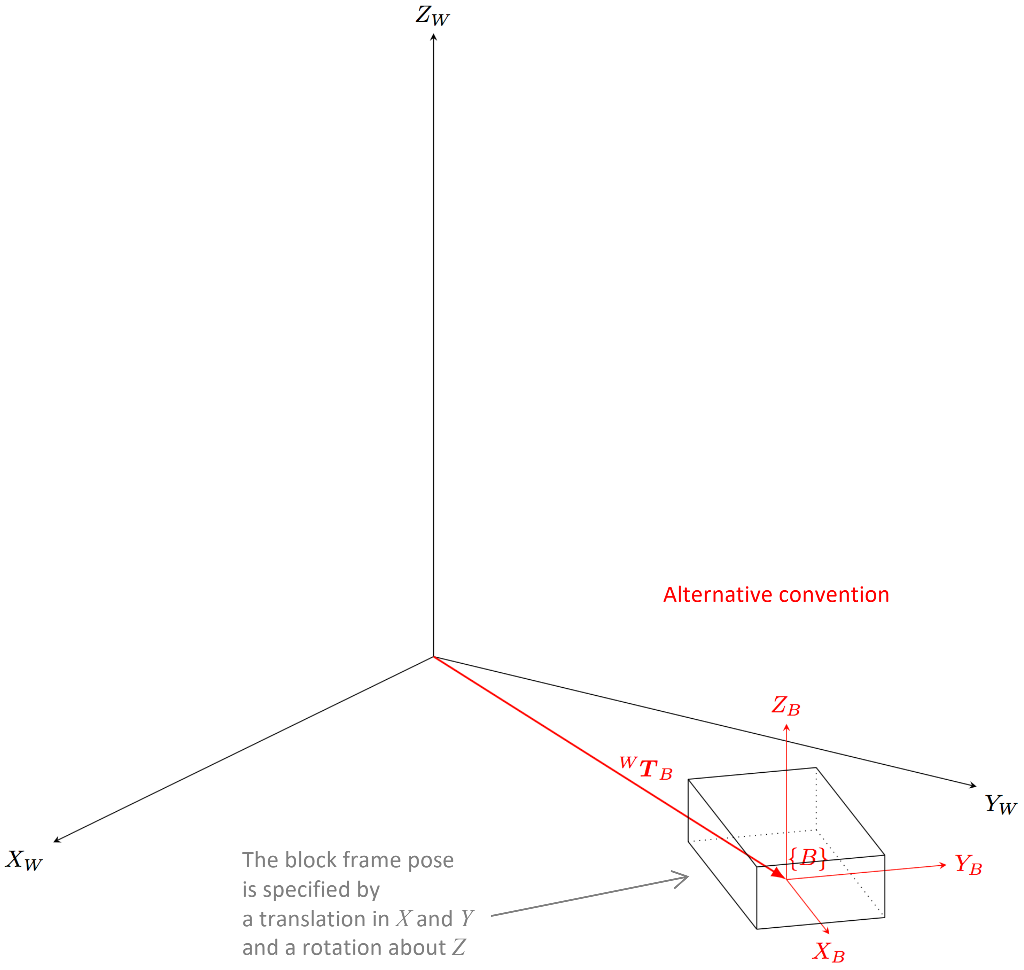
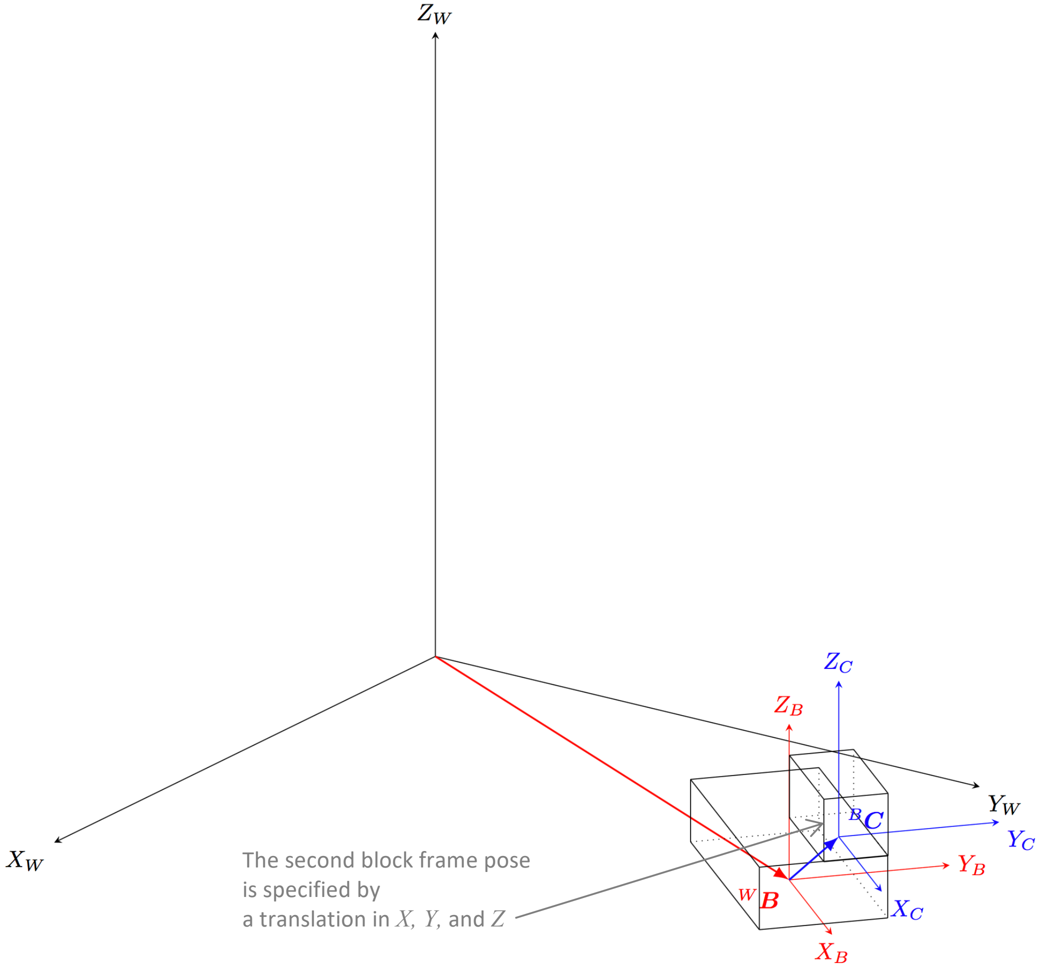


- We can arbitrarily position and orient one object, i.e. its pose, with respect to another object

- How? By specifying the required translations and rotations of its associated coordinate frame (homogeneous transformation)



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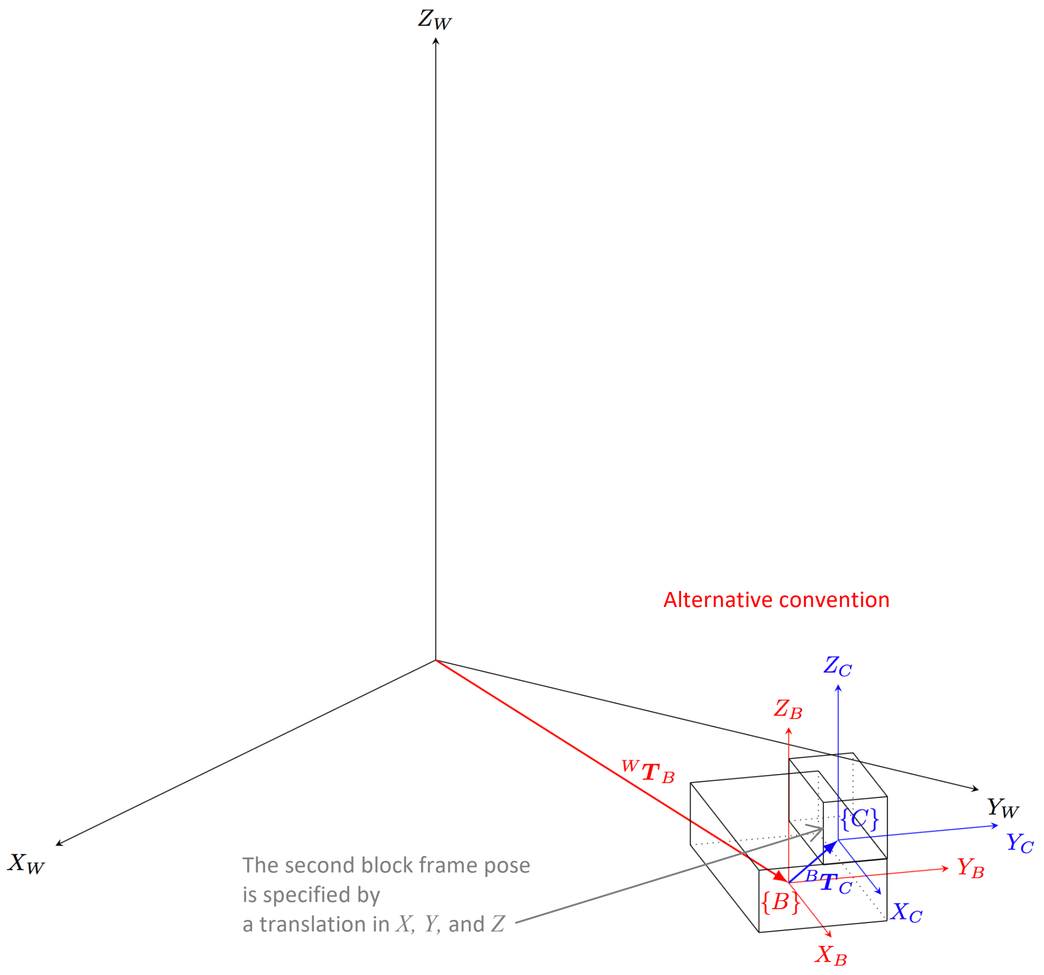


Specifying Pose in ROS

- We will use homogeneous transformations to specify a frame of reference, for end-effector and object pose

- ROS uses a different (but entirely equivalent) approach

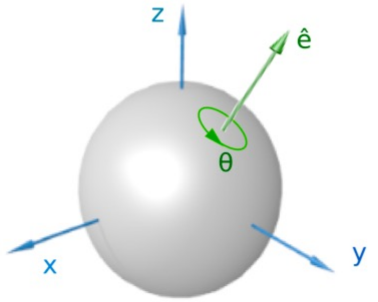
  – Specify the origin of the frame as a 3-D vector

  – Specify the orientation of the frame as a quaternion: a single rotation about some (appropriate) axis

Specifying Pose in ROS

- Euler's rotation theorem states that any displacement of a rigid body (in 3D space), such that a point on the rigid body remains fixed,

  is equivalent to a single rotation θ about some axis that runs through the fixed point



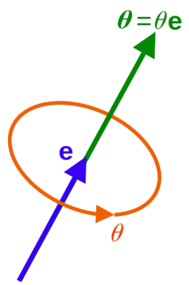
https://en.wikipedia.org/wiki/Euler%27s_rotation_theorem

- The axis of rotation is known as an Euler axis, typically represented by a unit vector **ê**

See: https://en.wikipedia.org/wiki/Rotation_formalisms_in_three_dimensions

Specifying Pose in ROS

- The product by $\theta\,\hat{\mathbf{e}}$ is known as an axis-angle



https://en.wikipedia.org/wiki/Axis–angle_representation

- Quaternions are a simple way to encode this axis–angle representation of a rotation in four numbers

Specifying Pose in ROS

- Quaternions are hypercomplex numbers

Vector part
(imaginary part)

$$q = w + \overbrace{x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}}$$

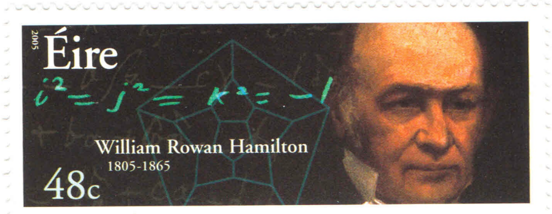
Scalar part
(real part)

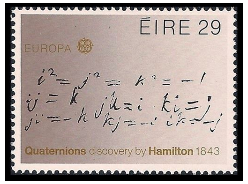
w, x, y, and z are real numbers

quaternion units

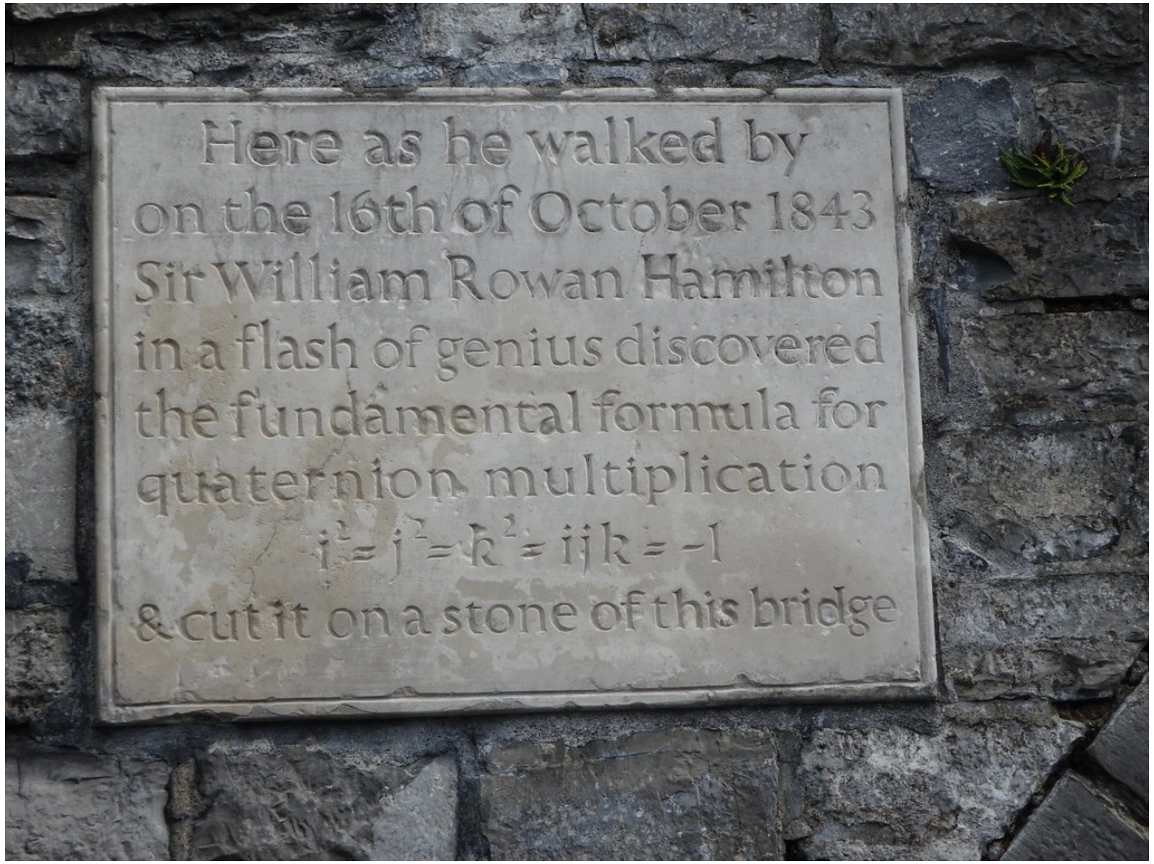
$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{i}\,\mathbf{j}\,\mathbf{k} = -1$$

- Discovered by Irish mathematician William Rowan Hamilton in 1843





Specifying Pose in ROS



Broom Bridge in Dublin, Ireland

https://www.flickr.com/photos/infomatique/44408785822

Specifying Pose in ROS

- A rotation of θ about the Euler axis $\hat{\mathbf{e}} = e_x\,\mathbf{i} + e_y\,\mathbf{j} + e_z\,\mathbf{k}$ is given by

$$
\begin{aligned}
q &= w + x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k} \\
&= \cos(\theta/2) + e_x \sin(\theta/2)\,\mathbf{i} + e_y \sin(\theta/2)\,\mathbf{j} + e_z \sin(\theta/2)\,\mathbf{k}
\end{aligned}
$$

NB: half the angle

- Equivalently

$$
q = \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}
$$

Specifying Pose in ROS

In ROS, we write it slightly differently

$$q = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

Axis part

Angle part

Specifying Pose in ROS

In CRAM (and Lisp), we write it the same was as ROS but as a list

$$(\underbrace{\mathtt{x}\ \ \mathtt{y}\ \ \mathtt{z}}_{\text{Axis part}}\ \ \mathtt{w})$$

Angle part

Specifying Pose in ROS

A rotation of -90° about the Z axis would be specified in quaternion notation as

$$q = \begin{pmatrix} 0.7071 \\ 0.0 \\ 0.0 \\ -0.7071 \end{pmatrix}$$

$$q = \begin{pmatrix} 0.0 \\ 0.0 \\ -0.7071 \\ 0.7071 \end{pmatrix}$$

ROS

CRAM (and Lisp)

```
(0.0 0.0 -0.7071 0.7071)
```

Recommended Reading

D. Vernon, Machine Vision – Automated Visual Inspection and Robot Vision, Prentice Hall International, 1991. Chapter 8.

http://vernon.eu/publications/91_Vernon_Machine_Vision.pdf

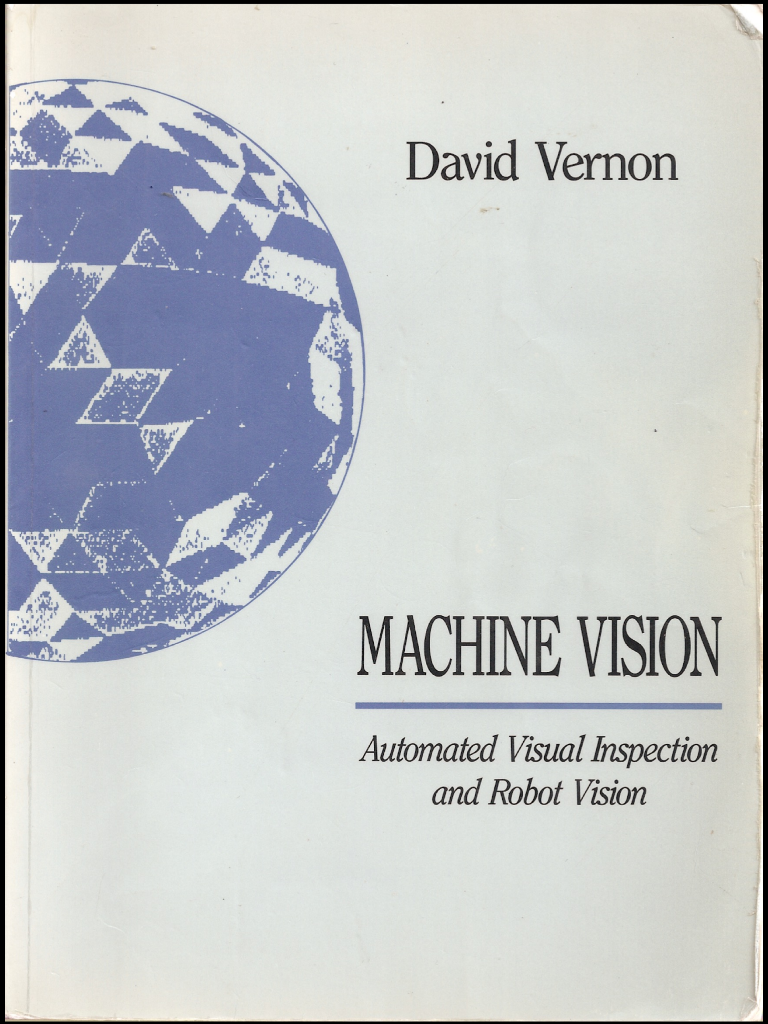
Similar material to that presented in this lecture.

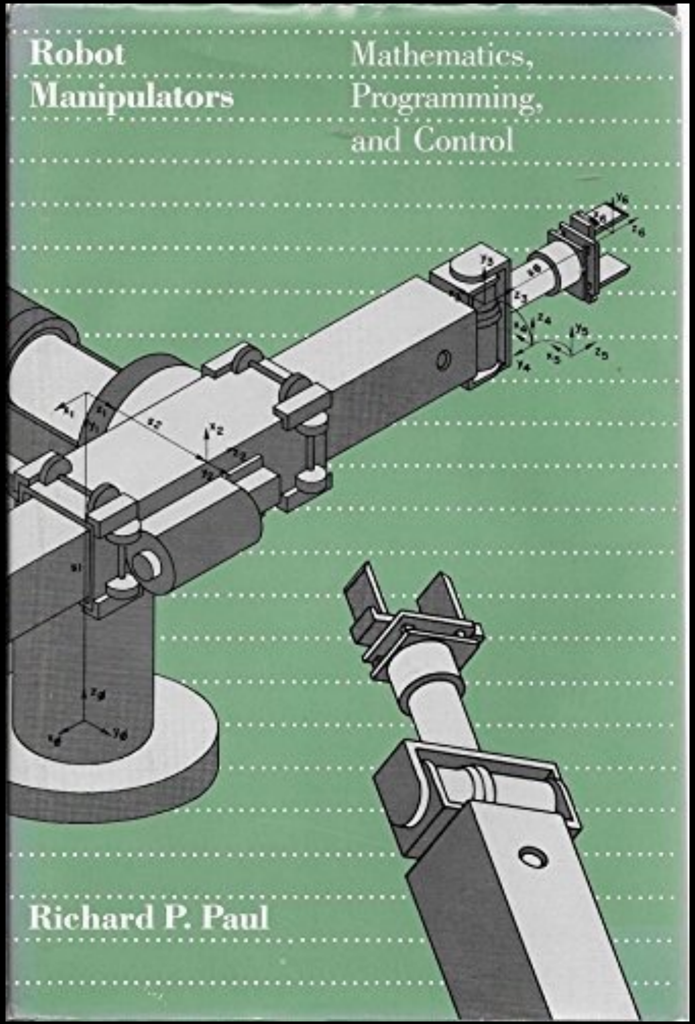
R. P. Paul, Robot Manipulators – Mathematics, Programming, and Control, MIT Press, 1981. Chapter 1.

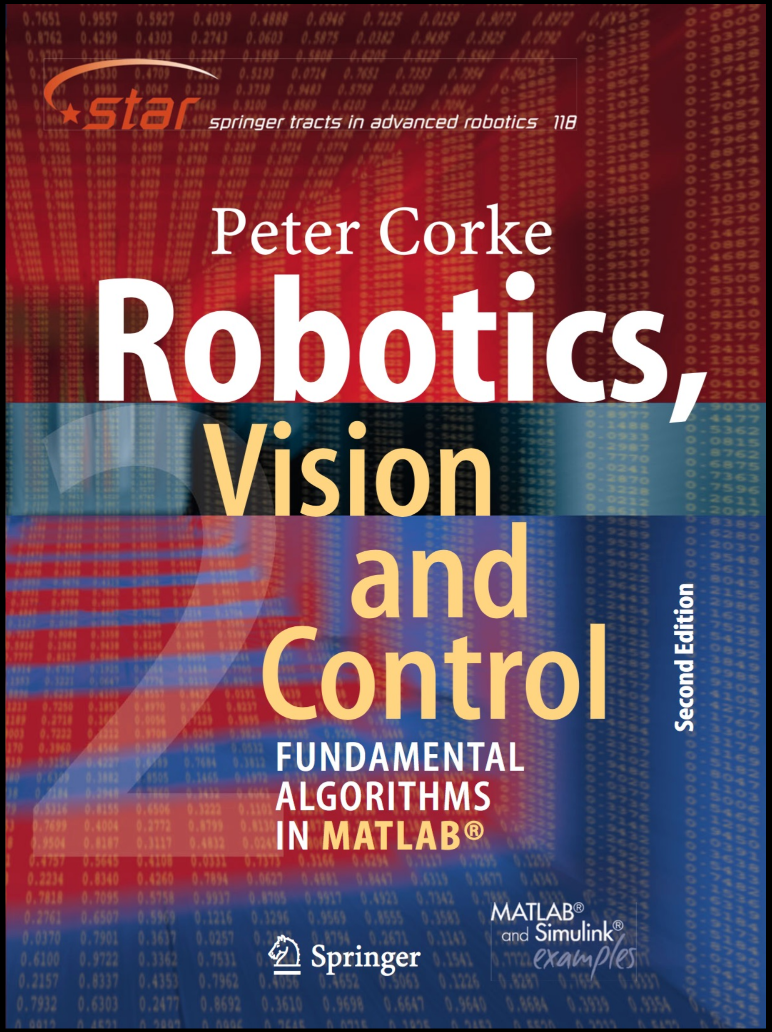
https://books.google.rw/books?id=UzZ3LAYqvRkC&printsec=frontcover&source=gbs_ViewAPI&redir_esc=y#v=onepage&q&f=false

Similar material to that presented in this lecture but complete comprehensive treatment.

P. Corke, Robotics, Vision and Control, 2nd Edition, Springer, 2017.
Comprehensive contemporary treatment; highly recommended.


David Vernon
MACHINE VISION
Automated Visual Inspection and Robot Vision


Robot Manipulators
Mathematics, Programming, and Control
Richard P. Paul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Peter Corke
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