Image Registration by Differential Evolution

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Abstract

In this paper we present the application of the differential evolution genetic algorithm to the task of image registration. Image registration is a fundamental task in image processing used to match two or more images. Genetic algorithms are a biologically inspired, stochastic search technique based on the principles of natural evolution, particularly "survival of the fittest", and have been found to be robust at searching large and complex search spaces.

In order to effect the registration of two images a transformation (called a spatial transformation) must be found which enables one image to be transformed with respect to the other, so that corresponding points in the two images will then be coincident. The spatial transformation is described by two polynomial warping functions and a fitness metric is used to quantify the error associated with a particular transformation. The metric used is the root mean square of the L1 distance over all control points. The differential evolution genetic algorithm of Price and Storn is a simple, effective and powerful genetic algorithm with few control parameters and is used in our case to minimise the fitness metric.

The efficacy of this approach is demonstrated with various data sets of increasing complexity including affine and rotational transformations. A quantitative comparison with least squared error solution is presented. Results show the genetic algorithm to be efficient and robust. In certain cases this approach significantly performs the least squares error solution.

1 Introduction

In this paper we present the application of the differential evolution genetic algorithm to the task of image registration.

Image registration is a fundamental task in image processing used to match two or more images. Genetic Algorithms (GAs) have been proved theoretically and empirically to be robust at searching large and complex spaces and have previously been applied to a wide range of optimisation problems including problems within the domain of computer vision [1] including specifically the registration problem [2] [3] [4] .

In this paper we apply the differential evolution genetic algorithm (DE) of Price and Storn to the task of image registration. The results presented demonstrate the efficacy of this approach with a variety of data sets including affine, non-linear and rotational transformations. In certain cases this approach significantly outperforms the least squares error solution.

2 Overview of Approach

2.1 Image Registration

"Registration is a fundamental task in image processing used to match two or more pictures taken, for example, at different times, or from different sensors, or form different viewpoints" [5]. In order to effect the registration of two images a transformation (called a spatial transformation) must be found which enables one image to be transformed with respect to the other, so that corresponding points in the two images will then be coincident.

The spatial transformation maps each point (x,y) in the output image to its corresponding point (i,j) in the input image :

$$(i,j) = (f_x(x,y), f_y(x,y)) \tag{1}$$

Both f_x and f_y are usually described by a polynomial function therefore :

$$(i,j) = (\sum_p^n \sum_q^n a_{pq} x^p y^q, \sum_p^n \sum_q^n b_{pq} x^p y^q) \tag{2}$$

Note, we choose n = 2 (which is adequate to correct for most distortions [6]), hence each polynomial has 9 coefficients.

Since the point (i,j) will usually not have integer co-ordinates, it is therefore necessary to either employ a nearest neighbour scheme or to interpolate amongst the pixels surrounding (i,j) in order to assign a grey level to (x,y).

Control points are points whose corresponding position in both images have been identified and therefore in effect they define the nature of the spatial transformation. Since the control points should be accurately mapped by the spatial transformation, they act as constraints which the spatial transformation should satisfy. Hence the error in mapping the control points can be used to quantify the fidelity of a particular transformation. The metric used to quantify this error is the root mean square of the L1 distance over all control points. It is necessary therefore, to find the polynomial coefficients of f_x and f_y which minimise this error.

The control points can of course be used to generate a set of simultaneous equations and a least squares error solution (LSE) can be then found as described in [7]. This work therefore sets out to find the polynomial coefficients using the differential evolution genetic algorithm.

2.2 Genetic Algorithms

Genetic Algorithms (GAs) are a biologically inspired, stochastic search technique based on the principles of natural evolution, particularly the notion of "survival of the fittest". Their origins lie in the work of John Holland at the University of Michigan during the seventies [8]. The goal of a GA is to find a point in a search space which maximises (or minimises) a particular objective function. The power of GAs lie in their robustness, they generally find an acceptable solution to a problem reasonably quickly, and they have been found to be robust at searching large and complex search spaces.

A GA maintains a population of strings which evolves over time. The values in a string are analogous to biological genes and each string encodes a point in the search space. The evaluation of the objective function at this point corresponds to the strings "fitness". This measure of fitness governs how the population evolves over time. During what's known as the reproductive phase, parent strings are selected and recombine to produce child strings which will makeup the next generation. Parents are selected randomly, with bias towards the selection of "fitter" parents. This leads to weak individuals dying out and fit individuals thriving and overall the population becomes fitter as a whole. In this way an optimal population member will ideally evolve.

2.3 Differential Evolution

The differential evolution genetic algorithm of Price and Storn [9] is a simple yet powerful genetic algorithm which has been previously used to tackle complex problems, the largest of which required the tuning of 60 parameters.

With DE a population of real valued solution vectors is randomly initialised so as to uniformly sample the search space. A single generation of DE involves the following. For each vector v1 in the current population, two other vectors v2 and v3 are randomly chosen and used along with a weighting factor F (usually in the range 0.4 - 1.2) to generate a difference vector v4. The vector v4 is then used to perturb v1 to produce v1'. If v1' is "fitter" than v1, then v1 is replaced by v1' in the population. This is repeated over a number of generations until either a pre-determined number of generations has passed or until the error associated with the best solution vector found to date is below a certain threshold value.

The main reason for the use of DE is the need for an extremely high quantization resolution over the parameter space, as is often the case with numerical optimisation problems. In our case parameters need to be able to take on values over 10 orders of magnitude (200 - 10^{-8}). This is most evident when considering the coefficients of the higher order polynomial terms in the spatial transformation which need to be able to take on extremely small values. This need for a high quantization resolution suggested using a floating point representation which is inherent in DE. Another advantage of using DE is its has relatively few control parameters.

3 Results

A software tool to enable the generation of sets of control points has been developed. This allows sets of 2-d grid points to be generated and spatially manipulated. A set of grid points is generated by spacing points over a 2-d grid (with dimensions 255 x 255). The second set of grid points is generated by transforming the first set as follows. A spatial transformation (as in equation 2) is first applied followed by an anti-clockwise rotation about the centre of the grid by an angle ϕ. Then a randomly generated value in the noise range is added to both the x and y co-ordinate of these points in order to perturb the transformation. This final addition of noise to the mapping ensures that there will be no exact solution to be found but instead a solution that minimises the error in mapping the control points.

Data Set	Noise Range	RMS error LSE	RMS error DEGA
1	1	0.3000	0.3000
2	5	2.467	2.4674
3	10	3.0103	3.0103
4	15	4.5155	4.5155
5	20	8.5315	8.5315
6	25	8.965	8.966
7	30	12.773	12.773
8	35	10.038	10.038
9	40	21.590	21.590

Table 1: Identity Transformation

Data Set	rot angle ϕ	Noise Range	RMS error LSE	RMS error DEGA
1	0.0	10	3.0103	3.0103
2	5.4	20	8.3283	8.5315
3	25	30	12.708	12.773
4	54	20	8.5635	8.5315
5	75	10	3.747	3.0103
6	72	20	8.1222	8.5315
7	100	25	8.859	8.9669
8	100	35	10.364	10.038
9	126	10	3.8455	3.0103
10	126	25	8.7436	8.5315

Table 2: Variety of Non-linear Transformations and Rotation by ϕ

Tables 1 and 2 present the results obtained. In table 1, all the data sets were generated by the identity transformation with noise in the noise as indicated. The data sets in table 2 are non-linear transformations. They were generated by varying the coefficients in the spatial transformation, combined with a rotation and the addition of random noise (as described above).

4 Discussion

The results presented demonstrate the efficacy of this approach with a variety of data sets including affine, non-linear and rotational transformations. In certain cases this approach significantly outperforms the least squares error solution.

The results presented in table 1 show DE to be able to match the performance of the LSE solution exactly under varying levels of noise. In table 2 where non-linear and rotational transformations transformations were used, DE was found to outperform the LSE solution in several cases. It must be pointed though that there were also cases where the LSE solution still performed better.

Also in the cases where DE outperforms the LSE solution, it was observed that the constant term in the polynomial (*i.e.* a_{00} coefficient) found by DE tended to have a relatively large value in comparison with the LSE solution constant term (*e.g.* 224.72 and 2.14 in data set no.5, table 2). Further investigation into this phenomenon is currently underway.

Note that in each case 400 generations of DE was used. It's recommended [9] that the population size be approximately equal to 10 times the number of parameters in the search space. In our case with nine polynomial coefficients to be found, a population size of 160 was found to be the most effective. A value of 0.4 for F, the weighting factor used in perturbing the population vectors, was determined empirically to be the most effective.

5 Conclusions

Results presented show the approach taken to be effective with a variety of data sets including affine, non-linear and rotational transformations. DE outperforms the least square error solution in several cases, while either matching or being slightly inferior in the remaining cases. It therefore acts as a useful and effective alternative approach to the LSE solution, which could be applied to a number of other domains particularly when a LSE solution isn't possible to obtain.

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School of Information and Software Engineering
Faculty of Informatics

Irish Machine Vision and Image Processing Conference (IMVIP-97)

and

Eight Ireland Conference on Artificial Intelligence (AI-97)

Volume 1