Removal of Superimposed Reflections from Dynamic Image Sequences

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Abstract

In this paper we show how secondary images superimposed on a primary dynamic image sequence can be separated to yield both primary and secondary images explicitly and distinctly. The problem is solved by analysing the Fourier spectrum of the additive images and by decoupling each spatial frequency component (which represents the composite image) into two distinct Fourier components, one for the primary image and one for the secondary reflected image. Specifically, we show how the individual frequency components which are associated with each moving object (or, rather, each set of objects moving with distinct constant velocity) can be computed from the single compound frequency magnitude and phase. Since the primary image and the second image exhibit different velocities, the decoupled components can then be sorted into the two sets (primary and secondary) on the basis of their phase change. This sorting is effected using a Hough transform. The original, separated, images are reconstructed using the inverse Fourier transform.

1 Introduction

In this paper, we are interested in 2-D functions which are the result of the superposition (addition) of two images: $\psi(x,y) = \psi^1(x,y) + \psi^2(x,y)$. Note, however, that a given component spatial frequency is likely to be present in both component images, even though each image may have a different velocity $(v^i_{x_k}, v^i_{y_k})$. Unfortunately, in the Fourier domain there is only one unique coefficient $\mathsf{F}(k_x, k_y) = \mathcal{F}(\psi(x,y))$ for each component $e^{-i(k_x x + k_y y)}$ rather than multiple distinct coefficients $\mathsf{F}^i(k_x, k_y)$ for each component image ψ^i. In the following section we will show that it is possible to decouple each Fourier component and to determine each $\mathsf{F}^i(k_x, k_y)$ from $\mathsf{F}(k_x, k_y)$ and we will identify the circumstances under which it is possible.

2 Decoupling Fourier Components

We note first that the conflated coefficient, *i.e.* the resultant Fourier component, is the sum of the Fourier components of each additive image: $\mathcal{F}(\psi) = \mathcal{F}\left(\sum_{i=1}^m \psi^i\right) = \sum_{i=1}^m \mathcal{F}(\psi^i)$. Equivalently, $\mathsf{F}(k_x, k_y) = \sum_{i=1}^m \mathsf{F}^i(k_x, k_y)$, for any given Fourier component k. The Fourier component $\mathsf{F}(k_x, k_y)$ is the vector sum of each $\mathsf{F}^i(k_x, k_y)$ and its resultant magnitude and phase will be dependent on the magnitudes and phases of the individual components $\mathsf{F}^i(k_x, k_y)$. Thus, for a given component, the Fourier transform only provides $\mathsf{F}(k_x, k_y)$, the composite coefficient, rather than what is required, *i.e.*, the coefficient corresponding to each image ψ^i. However, each $\mathsf{F}^i(k_x, k_y)$, can be determined from $\mathsf{F}(k_x, k_y)$.

Consider a 2-D function $\psi(x - v_x\delta t, y - v_y\delta t)$, which is $\psi(x,y)$ shifted by $(v_x\delta t, v_y\delta t)$, *i.e.*, it is travelling with constant velocity (v_x, v_y). The shift property of the Fourier transform states that the Fourier transform of any such shifted function is given by:

$$\begin{aligned} \mathcal{F}(\psi(x - v_x\delta t, y - v_y\delta t)) &= |\mathsf{F}(k_x, k_y)|\, e^{i\phi(k_x, k_y)} e^{-i(k_x v_x \delta t + k_y v_y \delta t)} \\ &= \mathcal{F}(\psi(x,y))\, e^{-i(k_x v_x \delta t + k_y v_y \delta t)} \end{aligned}$$

That is a spatial shift of a signal only produces a frequency-dependent phase change in the Fourier domain. The consequence of this is that the components of ψ^i translating with uniform velocity only undergo a phase change, with no change in magnitude; that is, the vector $\mathsf{F}^i(k_x, k_y)$ is only rotated by an angle given by the phase change $-i(k_x v_x^i \delta t + k_y v_y^i \delta t)$. Thus, the Fourier components at time t_n are related to those at time t_0 as follows:

$$\mathsf{F}^i_{t_n}(k_x, k_y) \quad = \quad \mathsf{F}^i_{t_0}(k_x, k_y)\left(e^{-i(k_x v_x^i \delta t + k_y v_y^i \delta t)}\right)^n \tag{1}$$

Also,

$$\mathsf{F}_{t_j}(k_x, k_y) = \sum_{i=1}^{m} \mathsf{F}^i_{t_j}(k_x, k_y) \tag{2}$$

where m is the number of individual images.

Rewriting equations 1 and 2, dropping the (k_x, k_y) for the sake of brevity while remembering that we are dealing with complex values defined on a 2-D domain, and letting $e^{-i(k_x v_x^i \delta t + k_y v_y^i \delta t)} = \Delta\Phi^i$, a complex variable representing the frequency- and velocity-dependent phase change, we have:

$$\mathsf{F}^i_{t_n} = \mathsf{F}^i_{t_0} \Delta\Phi^{in} \tag{3}$$

$$\mathsf{F}_{t_j} = \sum_{i=1}^{m} \mathsf{F}^i_{t_j} \tag{4}$$

Combining 3 and 4, we have

$$\mathsf{F}_{t_j} = \sum_{i=1}^{m} \mathsf{F}^i_{t_0} \Delta\Phi^{ij} \tag{5}$$

If we have $i = m$ distinct objects, then we have $2m$ complex unknowns (*i.e.* $\mathsf{F}^i_{t_0}$ and $\Delta\Phi^{ij}$) and consequently we can solve for these $2m$ unknowns if we have $2m$ constraints. These constraints are derived from equation 5 by making $j = 2m$ observations for F_{t_j}. That is, for a given spatial frequency (k_x, k_y), we observe from the Fourier transform F_{t_j} at time $t_0, t_1, \ldots, t_j, \ldots, t_{2m}$ and solve these $2m$ simultaneous equations of degree $2m - 1$ in complex unknowns $\mathsf{F}^i_{t_0}$ and $\Delta\Phi^i$.

Thus, we now have the decoupled Fourier transform components of each image and, furthermore, we have the phase change for each component of each object. Unfortunately, since we assigned the components arbitrarily, we do not (yet) know which component belongs to which image and we must now group them appropriately. That is, we have two sets of phase changes $\{\Delta\Phi^A\}$ and $\{\Delta\Phi^B\}$ and two corresponding sets of Fourier components $\{\mathsf{F}^A\}$ and $\{\mathsf{F}^B\}$ and we need to sort the elements of each set in to two new sets $\{\mathsf{F}^1\}$ and $\{\mathsf{F}^2\}$ corresponding to the two distinct images. To do this, we use our knowledge that there is a regularity in the temporal phase change as a function of frequency (k_x, k_y). Specifically, we have:

$$\begin{aligned} \Delta\Phi(k_x, k_y) &= e^{-i(v_x k_x \delta t + v_y k_y \delta t)} \\ &= e^{i\delta\phi(k_x, k_y)} \end{aligned}$$

For a given imagee i, (v_x^i, v_y^i) is constant. Thus a given object i will exhibit a phase change $\delta\phi(k_x, k_y)$:

$$\delta\phi(k_x, k_y) = -(v_x^i k_x \delta t + v_y^i k_y \delta t) \tag{6}$$

which will differ for each image i. Since we require $(v_x^i, v_y^i) \neq (v_x^j, v_y^j), i \neq j$, in order to sort the components of the two images we simply need to identify the two velocities (v_x^1, v_y^1) and (v_x^2, v_y^2) which will, in turn, allow us to identify the corresponding expected phase change for images 1 and 2, respectively. Let these expected phase changes be denoted $\delta\phi_e^1(k_x, k_y)$ and $\delta\phi_e^2(k_x, k_y)$, respectively. Then we assign a component $\mathsf{F}^A(k_x, k_y)$ to image 1, *i.e.* we include it in $\{\mathsf{F}^1\}$, if $\left|\delta\phi_e^1 - \delta\phi^A\right| < \left|\delta\phi_e^2 - \delta\phi^A\right|$, otherwise we assign it to $\{\mathsf{F}^2\}$; $\mathsf{F}^B(k_x, k_y)$ is assigned to the other image.

It only remains, then, to identify the two velocities (v_x^1, v_y^1) and (v_x^2, v_y^2). We do this using a Hough transform. From equation 6 we have:

$$v_y = \frac{1}{k_y \delta t}(\delta\phi(k_x, k_y) - k_x v_x \delta t) \tag{7}$$

This equation represents a Hough transform in the two variables (v_x, v_y) which we solve for all $\delta\phi(k_x, k_y), k_x, k_y$, and v_x. Note that $\delta\phi(k_x, k_y) = \arctan(\Im\Delta\Phi(k_x, k_y), \Re\Delta\Phi(k_x, k_y))$. Local maxima in this Hough transform space represent the velocities which are exhibited by the frequency components. In this case, there are two velocity maxima, one corresponding to image 1 and the other to image 2. The location of these maxima give us (v_x^1, v_y^1) and (v_x^2, v_y^2) and, thus, we can proceed to sort the components.

Note that the Hough transform equation 7 becomes degenerate if $k_y = 0$ in which case we use an alternative re-arrangement as follows:

$$v_x = \frac{(\delta\phi(k_x, k_y))}{k_x \delta t} \tag{8}$$

2.1 Limitations on the solution set of spatial frequencies

There are some practical limitations on this solution which relate to the velocities of the moving waves and their spatial frequencies. Specifically, there are limitations on the range of spatial frequencies for which the equations are valid and these limitations depend on the velocities of the moving images. We note here that this leads to the following constraint (for a proof of this, see [18]):

$$|k_x + k_y| \quad < \quad \frac{2\pi}{3\,|v^{max}|} \tag{9}$$

where $v_x^{max} = v_y^{max} = v^{max}$ and is the maximum velocity in either image. In effect, we can only solve for (or decouple) those spatial frequencies satisfying equation 9.

3 Results

Figures 1 and 2 demonstrate the application of the approach. Two images are moving independently of one another with velocities (v_{x_1}, v_{y_1}) and (v_{x_2}, v_{y_2}), respectively. The technique was tested for velocities in the ranges: $(0,0) \leq (v_{x_1}, v_{y_1}) \leq (0,5)$ and $(0.0) \leq (v_{x_2}, v_{y_2}) \leq (5.5)$, in increments of one pixel. Figures 1 and 2 depict each individual image at time t_0, the sum of both images at times t_0 and t_3, the result of computing (segmenting) the composite images based on the sequence at time t_0, t_1, t_2, and t_3, and the absolute value of the difference between the segmented images and the original images (scaled by a factor of 10), respectively.

Table 1 shows the RMS errors between the segmented image and the original image for all combinations of image velocities (in the range specified above).

4 Conclusions

The principal contribution of this paper is to show how the resultant Fourier component of a 2-D image comprising several additive signals, *i.e.* several distinct intensity waveforms, each translating (or shifting) with a distinct velocity can be resolved, or decoupled, into their respective components, with each resolved component corresponding to the distinct waveform or signal.

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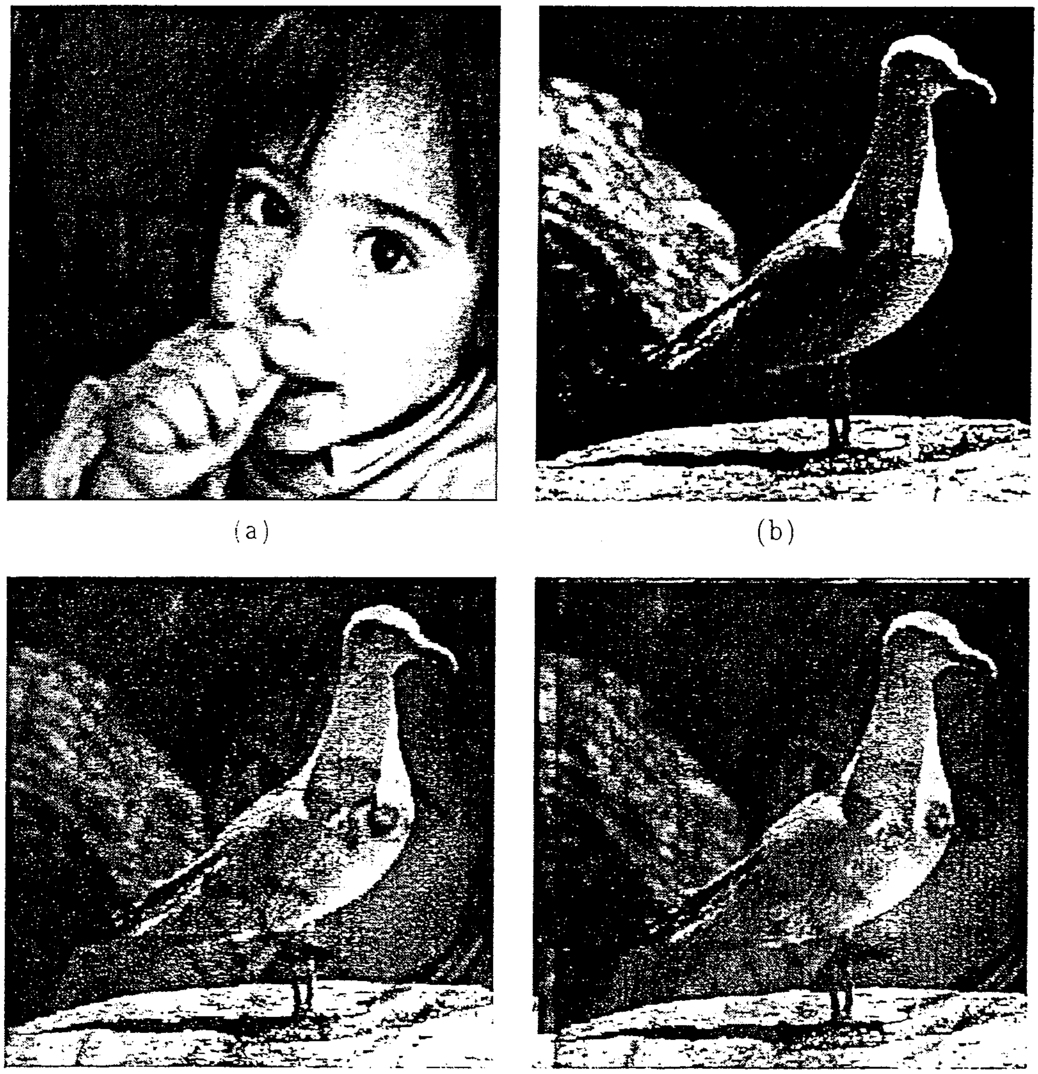
(v_{x_1}, v_{y_1})	RMS Error: Image 1 (velocity = (v_{x_1}, v_{y_1}))							RMS Error: Image 2 (velocity = (v_{x_2}, v_{y_2}))						
	v_{x_2}	v_{y_2}						v_{x_2}	v_{y_2}					
		0	1	2	3	4	5		0	1	2	3	4	5
(1, 0)	0	39.8	16.2	10.2	11.3	12.3	13.0	0	50.5	25.2	19.8	19.4	17.5	17.5
	1	49.6	26.3	44.8	43.3	43.6	45.9	1	67.0	30.2	52.1	55.0	57.2	59.4
	2	27.9	15.9	7.2	13.3	10.2	12.7	2	49.1	25.0	17.0	18.3	17.0	19.1
	3	30.0	9.1	15.7	42.6	38.6	41.0	3	44.2	16.8	24.2	50.8	48.5	52.3
	4	33.3	7.0	8.5	20.2	13.6	40.1	4	43.9	18.9	16.9	24.5	18.8	49.9
	5	25.6	9.7	13.3	12.8	39.3	41.5	5	43.7	19.0	17.9	18.9	45.5	47.3
(2, 0)	0	35.4	7.9	10.8	8.1	10.3	10.9	0	48.6	18.7	18.5	16.5	16.6	16.9
	1	39.5	14.0	9.5	29.9	30.9	30.0	1	50.6	23.4	18.4	38.4	37.4	37.2
	2	45.4	20.1	20.8	21.8	22.8	22.4	2	85.6	30.1	31.3	31.7	31.1	31.5
	3	35.5	18.6	15.3	10.9	38.3	37.8	3	47.1	25.7	25.5	16.3	45.2	46.8
	4	29.1	9.1	18.6	10.7	12.3	14.1	4	44.9	17.1	24.6	18.1	19.5	19.1
	5	33.2	10.1	10.5	20.9	14.2	42.3	5	43.3	18.7	18.3	27.0	19.9	44.5
(3, 0)	0	35.5	13.3	9.4	28.1	11.8	11.8	0	47.0	19.3	17.6	28.0	16.9	16.8
	1	35.4	10.8	11.4	35.1	40.9	40.6	1	47.0	19.1	20.0	42.6	47.5	49.4
	2	32.0	25.7	10.4	10.5	12.7	13.5	2	50.7	29.7	20.5	17.4	18.4	17.5
	3	51.8	28.5	23.3	44.9	45.3	43.4	3	59.5	31.2	29.8	56.2	56.1	55.1
	4	36.0	15.4	9.9	14.8	12.5	14.6	4	48.0	22.9	17.4	18.5	18.8	19.5
	5	27.9	8.9	16.8	13.4	42.8	42.7	5	44.8	18.1	23.3	19.8	46.3	47.6
(4, 0)	0	36.4	7.5	12.1	10.6	13.7	12.8	0	47.8	16.7	18.6	16.7	19.5	17.1
	1	37.0	10.4	11.4	28.5	28.7	29.4	1	47.6	18.9	19.7	33.0	36.8	35.3
	2	36.1	10.3	16.2	11.0	11.1	12.5	2	47.5	18.3	23.5	17.4	17.1	17.1
	3	33.8	12.8	12.4	13.9	30.1	29.9	3	49.0	24.3	19.1	19.9	39.8	37.8
	4	46.0	21.4	21.3	22.2	22.5	24.3	4	77.7	29.1	29.6	30.0	29.4	30.2
	5	36.9	19.2	15.7	12.8	15.3	38.4	5	47.8	26.0	25.1	17.9	19.6	44.8
(5, 0)	0	36.2	8.7	9.8	11.2	13.3	21.3	0	47.7	18.2	17.9	17.4	18.1	23.2
	1	36.2	10.6	13.1	37.5	39.4	40.4	1	47.7	18.2	20.5	48.8	48.5	46.8
	2	36.2	11.2	11.7	27.0	12.6	14.2	2	47.7	19.6	19.3	30.1	18.7	18.3
	3	36.2	14.4	18.0	12.5	40.9	44.0	3	47.7	22.3	27.8	19.9	48.6	48.3
	4	31.8	18.6	12.7	14.1	13.6	16.7	4	51.4	29.3	20.5	21.1	18.9	19.3
	5	44.1	28.0	23.7	23.0	23.7	44.9	5	66.0	32.6	30.3	31.1	30.1	56.3

Table 1: RMS error for additive child and gull images; image 1 (child) velocity = (v_{x_1}, v_{y_1}) pixels, image 2 (gull) velocity = (v_{x_2}, v_{y_2}) pixels.



(a) (b) (c) (d)

Figure 1: Segmentation test scenario 1: (a) and (b) images 1 and 2 translating with velocities (0, 4) and (1, 1) pixels, respectively; (c) and (d) sum of images 1 and 2 at time t_0 and t_3.

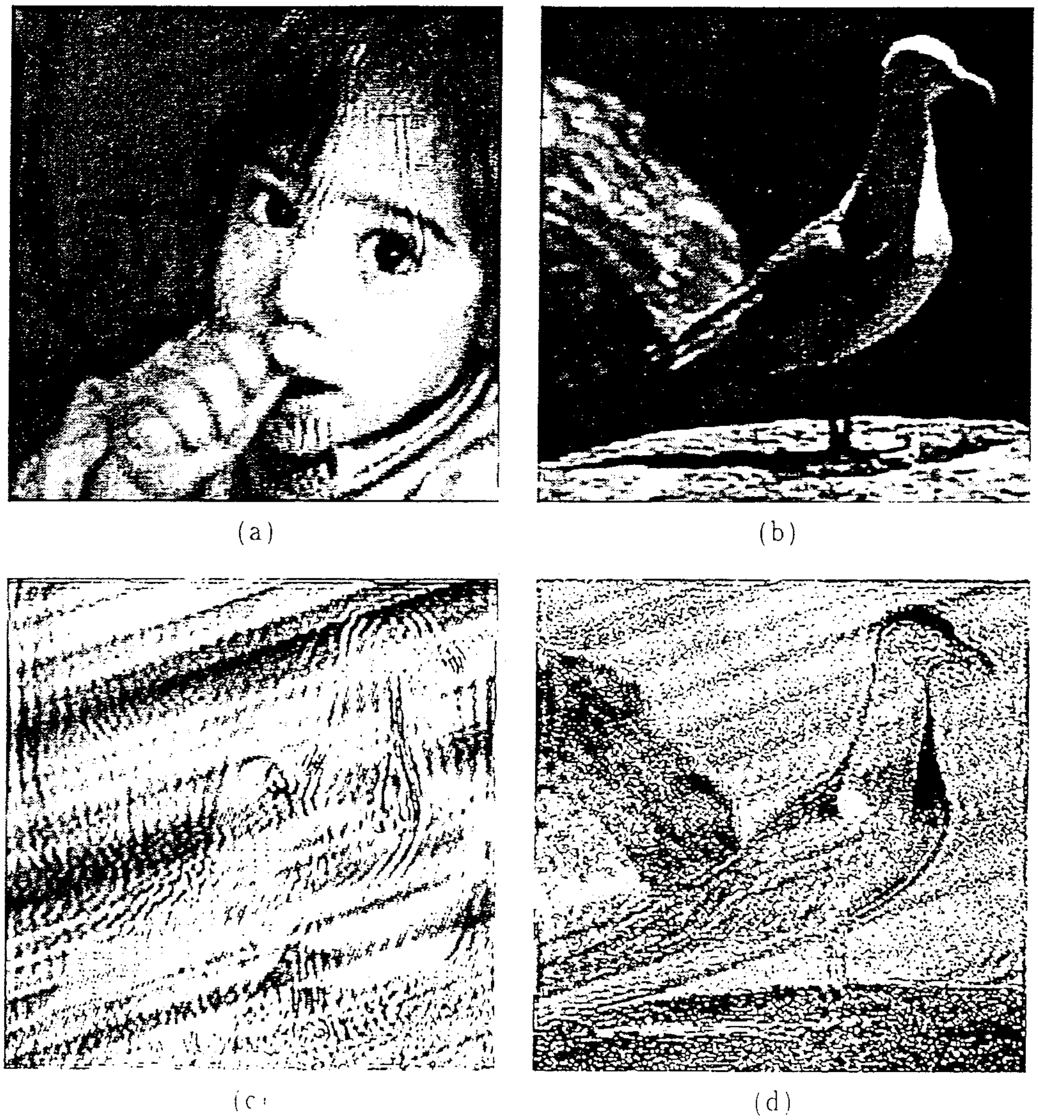


Figure 2: Segmentation test scenario 1: (a) and (b) the result of computing (segmenting) the composite images based on the sequence at times t_0, t_1, t_2, and t_3: (c) and (d) the absolute value of the difference between the segmented images and the original images (scaled by a factor of 10).

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